Control of Multiple-Input, Multiple-Output (MIMO) Processes

18.1 Process Interactions and Control Loop Interactions
18.2 Pairing of Controlled and Manipulated Variables
18.3 Singular Value Analysis
18.4 Tuning of Multiloop PID Control Systems
18.5 Decoupling and Multivariable Control Strategies
18.6 Strategies for Reducing Control Loop Interactions
Control of Multivariable Processes

In practical control problems there typically are a number of process variables which must be controlled and a number which can be manipulated.

Example: product quality and throughput must usually be controlled.

Several simple physical examples are shown in Fig. 18.1.

Note the "process interactions" between controlled and manipulated variables.
Figure 18.1. Physical examples of multivariable control problems.
Figure 18.2. SISO and MIMO control problems.
• Controlled Variables: \( x_D, x_B, P, h_D, \) and \( h_B \)

• Manipulated Variables: \( D, B, R, Q_D, \) and \( Q_B \)

Note: Possible multiloop control strategies = \( 5! = 120 \)
In this chapter we will be concerned with characterizing process interactions and selecting an appropriate multiloop control configuration.

If process interactions are significant, even the best multiloop control system may not provide satisfactory control.

In these situations there are incentives for considering multivariable control strategies.

**Definitions:**

- **Multiloop control:** Each manipulated variable depends on only a single controlled variable, i.e., a set of conventional feedback controllers.

- **Multivariable Control:** Each manipulated variable can depend on two or more of the controlled variables.

**Examples:** decoupling control, model predictive control
Multiloop Control Strategy

- Typical industrial approach
- Consists of using $n$ standard FB controllers (e.g., PID), one for each controlled variable.

- Control system design
  1. Select controlled and manipulated variables.
  2. Select pairing of controlled and manipulated variables.
  3. Specify types of FB controllers.

Example: 2 x 2 system

Two possible controller pairings:

\[ U_1 \text{ with } Y_1, \ U_2 \text{ with } Y_2 \quad (1-1/2-2 \text{ pairing}) \]

or

\[ U_1 \text{ with } Y_2, \ U_2 \text{ with } Y_1 \quad (1-2/2-1 \text{ pairing}) \]

Note: For $n \times n$ system, $n!$ possible pairing configurations.
Transfer Function Model (2 x 2 system)

Two controlled variables and two manipulated variables
(4 transfer functions required)

\[
\frac{Y_1(s)}{U_1(s)} = G_{p11}(s), \quad \frac{Y_2(s)}{U_2(s)} = G_{p12}(s) \tag{18-1}
\]

\[
\frac{Y_2(s)}{U_1(s)} = G_{p21}(s), \quad \frac{Y_2(s)}{U_2(s)} = G_{p22}(s)
\]

Thus, the input-output relations for the process can be written as:

\[
Y_1(s) = G_{p11}(s)U_1(s) + G_{p12}(s)U_2(s) \tag{18-2}
\]

\[
Y_2(s) = G_{p21}(s)U_1(s) + G_{p22}(s)U_2(s) \tag{18-3}
\]
Or in vector-matrix notation as,

\[ Y(s) = G_p(s)U(s) \]  \hspace{1cm} (18–4)

where \( Y(s) \) and \( U(s) \) are vectors,

\[
Y(s) = \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} \quad U(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \]  \hspace{1cm} (18–5)

And \( G_p(s) \) is the transfer function matrix for the process

\[
G_p(s) = \begin{bmatrix} G_{P11}(s) & G_{P12}(s) \\ G_{P21}(s) & G_{P22}(s) \end{bmatrix} \]  \hspace{1cm} (18–6)
Figure 18.3. Block diagrams for 2 X 2 multiloop control schemes
Control-loop Interactions

• Process interactions may induce undesirable interactions between two or more control loops.

  Example: 2 x 2 system
  Control loop interactions are due to the presence of a third feedback loop.

• Problems arising from control loop interactions
  i. Closed-loop system may become destabilized.
  ii. Controller tuning becomes more difficult.
Figure 18.4. The hidden feedback control loop (in dark lines) for a 1-1/2-2 controller pairing.
Figure 18.5 Set point responses for Example 18.1 using ITAE tuning.
Block Diagram Analysis

For the multiloop control configuration, the transfer function between a controlled and a manipulated variable depends on whether the other feedback control loops are open or closed.

**Example: 2 x 2 system, 1-1/2 -2 pairing**

From block diagram algebra we can show

\[
\frac{Y_1(s)}{U_1(s)} = G_{P11}(s), \quad \text{(second loop open)} \quad (18-7)
\]

\[
\frac{Y_1(s)}{U_1(s)} = G_{P11} - \frac{G_{P12}G_{P21}G_{C2}}{1 + G_{C2}G_{P22}} \quad \text{(second loop closed)} \quad (18-11)
\]

Note that the last expression contains \( G_{C2} \).
18.1.2 Closed-Loop Stability

To evaluate the effects of control loop interactions further, again consider the block diagram for the 1-1/2-2 control scheme in Fig. 18.3a. Using block diagram algebra (see Chapter 11), we can derive the following expressions relating controlled variables and set points:

\[
Y_1 = \Gamma_{11} Y_{sp1} + \Gamma_{12} Y_{sp2} \quad (18-13)
\]
\[
Y_2 = \Gamma_{21} Y_{sp1} + \Gamma_{22} Y_{sp2} \quad (18-14)
\]

where the closed-loop transfer functions are

\[
\Gamma_{11} = \frac{G_c G_{p11} + G_c G_c (G_{p11} G_{p22} - G_{p12} G_{p21})}{\Delta(s)} \quad (18-15)
\]
\[
\Gamma_{12} = \frac{G_c G_{p12}}{\Delta(s)} \quad (18-16)
\]
\[
\Gamma_{21} = \frac{G_c G_{p21}}{\Delta(s)} \quad (18-17)
\]
\[
\Gamma_{22} = \frac{G_c G_{p22} + G_c G_c (G_{p11} G_{p22} - G_{p12} G_{p21})}{\Delta(s)} \quad (18-18)
\]

and \(\Delta(s)\) is defined as

\[
\Delta(s) = (1 + G_c G_{p11})(1 + G_c G_{p22}) - G_c G_c G_{p12} G_{p21} \quad (18-19)
\]

Two important conclusions can be drawn from these closed-loop relations. First, a set-point change in one loop causes both controlled variables to change because \(\Gamma_{12}\) and \(\Gamma_{21}\) are not zero, in general. The second conclusion concerns the stability of the closed-loop system. Because each of the four closed-loop transfer functions in Eqs. 18-15 to 18-18 has the same denominator, the characteristic equation is \(\Delta(s) = 0\), or

\[
(1 + G_c G_{p11})(1 + G_c G_{p22}) - G_c G_c G_{p12} G_{p21} = 0 \quad (18-20)
\]

Thus, the stability of the closed-loop system depends on both controllers, \(G_c\) and \(G_c\), and all four process transfer functions. An analogous characteristic equation can be derived for the 1-2/2-1 control scheme in Fig. 18.3b.

For the special case where either \(G_{p12} = 0\) or \(G_{p21} = 0\), the characteristic equation in Eq. 18-20 reduces to

\[
(1 + G_c G_{p11})(1 + G_c G_{p22}) = 0 \quad (18-21)
\]

For this situation, the stability of the overall system merely depends on the stability of the two individual feedback control loops and their characteristic equations.

\[
1 + G_c G_{p11} = 0 \quad \text{and} \quad 1 + G_c G_{p22} = 0 \quad (18-22)
\]
EXAMPLE 18.2

Consider a process that can be described by the transfer function matrix (Gagnepain and Seborg, 1982):

\[
G_p(s) = \begin{bmatrix}
\frac{2}{10s + 1} & \frac{1.5}{s + 1} \\
\frac{1.5}{s + 1} & \frac{2}{10s + 1}
\end{bmatrix}
\]

Assume that two proportional feedback controllers are to be used so that \( G_{c1} = K_{c1} \) and \( G_{c2} = K_{c2} \). Determine the values of \( K_{c1} \) and \( K_{c2} \) that result in closed-loop stability for both the 1-1/2-2 and 1-2/2-1 configurations.

**SOLUTION**

The characteristic equation for the closed-loop system is obtained by substitution into Eq. 18-20 and collecting powers of \( s \) as follows:

\[
a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 = 0
\]

where

\[
a_4 = 100
\]

\[
a_3 = 20K_{c1} + 20K_{c2} + 220
\]

\[
a_2 = 42K_{c1} + 42K_{c2} - 221K_{c1}K_{c2} + 141
\]

\[
a_1 = 24K_{c1} + 24K_{c2} + 8K_{c1}K_{c2} + 22
\]

\[
a_0 = 2K_{c1} + 2K_{c2} + 1.75K_{c1}K_{c2} + 1
\]

Note that the characteristic equation in (18-23) is fourth order, even though each individual transfer function in \( G_p(s) \) is first order.

The controller gains that result in a stable closed-loop system can be determined by applying the Routh stability criterion (Chapter 11) for specified values of \( K_{c1} \) and \( K_{c2} \). The resulting stability regions are shown in Fig. 18.6. If either \( K_{c1} \) or \( K_{c2} \) is close to zero, the other controller gain can be an arbitrarily large, positive value and still have a stable closed-loop system. This result is a consequence of having process transfer functions that are first order without time delay, which is an idealistic case. MIMO control systems normally have an upper bound for stability for both controller gains for all values of \( K_{c1} \).

A similar stability analysis can be performed for the 1-2/2-1 control configuration. The calculated stability regions are shown in Fig. 18.7. A comparison of Figs. 18.6 and 18.7 indicates that the 1-2/2-1 control scheme results in a larger stability region because a wider range of controller gains can be used. For example, suppose that \( K_{c1} = 2 \). Then Fig. 18.6 indicates that the 1-1/2-2
Figure 18.6. Stability region for Example 18.2 with 1-1/2-2 controller pairing
Figure 18.7. Stability region for Example 18.2 with 1-2/2-1 controller pairing
Relative Gain Array

• Provides two types of useful information:
  1. Measure of process interactions
  2. Recommendation about best pairing of controlled and manipulated variables.

• Requires knowledge of steady-state gains but not process dynamics.
Example of RGA Analysis: 2 x 2 system

- Steady-state process model,

\[ y_1 = K_{11}u_1 + K_{12}u_2 \]
\[ y_2 = K_{21}u_1 + K_{22}u_2 \]

The RGA, \( \Lambda \), is defined as:

\[ \Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \]

where the relative gain, \( \lambda_{ij} \), relates the \( i^{th} \) controlled variable and the \( j^{th} \) manipulated variable

\[ \lambda_{ij} \triangleq \frac{\left( \frac{\partial y_i}{\partial u_j} \right)_u}{\left( \frac{\partial y_i}{\partial u_j} \right)_y} = \frac{\text{open-loop gain}}{\text{closed-loop gain}} \quad (18-24) \]
Scaling Properties:

i. $\lambda_{ij}$ is dimensionless

ii. $\sum_i \sum_j \lambda_{ij} = 1.0$

For a 2 x 2 system,

$$\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}, \quad \lambda_{12} = 1 - \lambda_{11} = \lambda_{21} \quad (18-34)$$

Recommended Controller Pairing

It corresponds to the $\lambda_{ij}$ which have the largest positive values that are closest to one.
**In general:**

1. Pairings which correspond to negative pairings should not be selected.
2. Otherwise, choose the pairing which has $\lambda_{ij}$ closest to one.

**Examples:**

<table>
<thead>
<tr>
<th>Process Gain Matrix, $K$</th>
<th>Relative Gain Array, $\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\begin{bmatrix} K_{11} &amp; 0 \ 0 &amp; K_{22} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\begin{bmatrix} 0 &amp; K_{12} \ K_{21} &amp; 0 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\begin{bmatrix} K_{11} &amp; K_{12} \ 0 &amp; K_{22} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
</tr>
<tr>
<td>$\begin{bmatrix} K_{11} &amp; 0 \ K_{21} &amp; K_{22} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
For 2 x 2 systems:

\[
y_1 = K_{11}u_1 + K_{12}u_2
\]

\[
y_2 = K_{21}u_1 + K_{22}u_2
\]

\[
\lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}, \quad \lambda_{12} = 1 - \lambda_{11} = \lambda_{21}
\]

Example 1:

\[
K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1.5 \\ 1.5 & 2 \end{bmatrix}
\]

\[
A = \begin{bmatrix} 2.29 & -1.29 \\ -1.29 & 2.29 \end{bmatrix}
\]

\[
\therefore \quad \text{Recommended pairing is } Y_1 \text{ and } U_1, Y_2 \text{ and } U_2.
\]

Example 2:

\[
K = \begin{bmatrix} -2 & 1.5 \\ 1.5 & 2 \end{bmatrix} \Rightarrow \quad A = \begin{bmatrix} 0.64 & 0.36 \\ 0.36 & 0.64 \end{bmatrix}
\]

\[
\therefore \quad \text{Recommended pairing is } Y_1 \text{ with } U_1 \text{ and } Y_2 \text{ with } U_2.
\]
The RGA can be expressed in two equivalent forms:

\[
\begin{align*}
W &= \begin{bmatrix} W_h & W_c \\ T - T_c & T - T_c \\ T_h - T & T_h - T_c \end{bmatrix} \\
K &= \begin{bmatrix} \frac{W_h}{T_h - T_c} & \frac{W_c}{T - T_c} \\ \frac{T_h - T}{T - T_c} & \frac{T_h - T_c}{T - T} \end{bmatrix} \\
\Lambda &= \begin{bmatrix} \frac{W_h}{W_c + W_h} & \frac{W_c}{W_c + W_h} \\ \frac{W_h}{W_c + W_h} & \frac{W_c}{W_c + W_h} \end{bmatrix}
\end{align*}
\]

Note that each relative gain is between 0 and 1. The recommended controller pairing depends on nominal values of \( T, T_h, \) and \( T_c. \)
RGA for Higher-Order Systems

For and \( n \times n \) system,

\[
\begin{pmatrix}
    u_1 & u_2 & \cdots & u_n \\
    y_1 & \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\
    y_2 & \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    y_n & \lambda_{n1} & \lambda_{n1} & \cdots & \lambda_{nn}
\end{pmatrix}
\]

Each \( \lambda_{ij} \) can be calculated from the relation,

\[
\lambda_{ij} = K_{ij} H_{ij} \quad (18-37)
\]

where \( K_{ij} \) is the \((i,j)\)-element of the steady-state gain \( K \) matrix,

\[
y = Ku
\]

\( H_{ij} \) is the \((i,j)\)-element of the \( H = \left(K^{-1}\right)^T \)

Note :

\( \Lambda \neq KH \)
Example: Hydrocracker

The RGA for a hydrocracker has been reported as,

\[
\Lambda = \begin{bmatrix}
    y_1 & u_1 & u_2 & u_3 & u_4 \\
    0.931 & 0.150 & 0.080 & -0.164 \\
    y_2 & -0.011 & -0.429 & 0.286 & 1.154 \\
    y_3 & -0.135 & 3.314 & -0.270 & -1.910 \\
    y_4 & 0.215 & -2.030 & 0.900 & 1.919
\end{bmatrix}
\]

Recommended controller pairing?
Singular Value Analysis

• Any real \( m \times n \) matrix can be factored as,
  \[ K = W \Sigma V^T \]

• Matrix \( \Sigma \) is a diagonal matrix of singular values:
  \[ \Sigma = \text{diag} (\sigma_1, \sigma_2, \ldots, \sigma_r) \]

• The singular values are the positive square roots of the eigenvalues of \( K^T K \) (\( r \) = the rank of \( K^T K \)).

• The columns of matrices \( W \) and \( V \) are orthonormal. Thus,
  \[ WW^T = I \quad \text{and} \quad VV^T = I \]

• Can calculate \( \Sigma, W, \) and \( V \) using MATLAB command, \texttt{svd}.

• Condition number \((CN)\) is defined to be the ratio of the largest to the smallest singular value,
  \[ CN \triangleq \frac{\sigma_1}{\sigma_r} \]

• A large value of \( CN \) indicates that \( K \) is ill-conditioned.
Condition Number

• CN is a measure of sensitivity of the matrix properties to changes in individual elements.
• Consider the RGA for a 2x2 process,

\[ K = \begin{bmatrix} 1 & 0 \\ 10 & 1 \end{bmatrix} \Rightarrow \Lambda = I \]

• If \( K_{12} \) changes from 0 to 0.1, then \( K \) becomes a singular matrix, which corresponds to a process that is difficult to control.
• RGA and SVA used together can indicate whether a process is easy (or difficult) to control.

\[ \Sigma(K) = \begin{bmatrix} 10.1 & 0 \\ 0 & 0.1 \end{bmatrix} \quad \text{CN} = 101 \]

• \( K \) is poorly conditioned when CN is a large number (e.g., \( > 10 \)). Thus small changes in the model for this process can make it very difficult to control.
### Selection of Inputs and Outputs

- Arrange the singular values in order of largest to smallest and look for any $\frac{\sigma_i}{\sigma_{i-1}} > 10$; then one or more inputs (or outputs) can be deleted.

- Delete one row and one column of $K$ at a time and evaluate the properties of the reduced gain matrix.

- **Example:**

$$K = \begin{bmatrix}
0.48 & 0.90 & -0.006 \\
0.52 & 0.95 & 0.008 \\
0.90 & -0.95 & 0.020 
\end{bmatrix}$$
\[ W = \begin{bmatrix} 0.5714 & 0.3766 & 0.7292 \\ 0.6035 & 0.4093 & -0.6843 \\ -0.5561 & 0.8311 & 0.0066 \end{bmatrix} \]

\[ \Sigma = \begin{bmatrix} 1.618 & 0 & 0 \\ 0 & 1.143 & 0 \\ 0 & 0 & 0.0097 \end{bmatrix} \]

\[ V = \begin{bmatrix} 0.0541 & 0.9984 & 0.0151 \\ 0.9985 & -0.0540 & -0.0068 \\ -0.0060 & 0.0154 & -0.9999 \end{bmatrix} \]

\[ \Lambda = \begin{bmatrix} -2.4376 & 3.0241 & 0.4135 \\ 1.2211 & -0.7617 & 0.5407 \\ 2.2165 & -1.2623 & 0.0458 \end{bmatrix} \]

Preliminary pairing: \( y_1-u_2, y_2-u_3, y_3-u_1 \).

CN suggests only two output variables can be controlled. Eliminate one input and one output (3x3\( \rightarrow \)2x2).

CN = 166.5 \( (\sigma_1/\sigma_3) \)
Question:

How sensitive are these results to the scaling of inputs and outputs?

<table>
<thead>
<tr>
<th>Pairing Number</th>
<th>Controlled Variables</th>
<th>Manipulated Variables</th>
<th>CN</th>
<th>λ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>y₁, y₂</td>
<td>u₁, u₂</td>
<td>184</td>
<td>39.0</td>
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<tr>
<td>2</td>
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<td>72.0</td>
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<td>u₁, u₃</td>
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<td>3.25</td>
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<td>9</td>
<td>y₂, y₃</td>
<td>u₂, u₃</td>
<td>67.9</td>
<td>0.714</td>
</tr>
</tbody>
</table>
Alternative Strategies for Dealing with Undesirable Control Loop Interactions

1. "Detune" one or more FB controllers.
2. Select different manipulated or controlled variables. e.g., nonlinear functions of original variables
3. Use a decoupling control scheme.
4. Use some other type of multivariable control scheme.

Decoupling Control Systems

- **Basic Idea**: Use additional controllers to compensate for process interactions and thus reduce control loop interactions

- Ideally, decoupling control allows setpoint changes to affect only the desired controlled variables.

- Typically, decoupling controllers are designed using a simple process model (e.g., a steady-state model or transfer function model)
Figure 18.9 A decoupling control system.
Decoupler Design Equations

We want cross-controller, $T_{12}$, to cancel the effect of $U_2$ on $Y_1$. Thus, we would like,

$$T_{12} G_{P11} U_{22} + G_{P12} U_{22} = 0 \quad (18-79)$$

Because $U_{22} \neq 0$ in general, then

$$T_{12} = -\frac{G_{P12}}{G_{P11}} \quad (18-80)$$

Similarly, we want $T_{12}$ to cancel the effect of $U_1$ on $Y_2$. Thus, we require that,

$$T_{21} G_{P22} U_{11} + G_{P21} U_{11} = 0 \quad (18-76)$$

$$\therefore T_{21} = -\frac{G_{P21}}{G_{P22}} \quad (18-78)$$

Compare with the design equations for feedforward control based on block diagram analysis.
Variations on a Theme

1. **Partial Decoupling:**
   Use only one “cross-controller.”

2. **Static Decoupling:**
   Design to eliminate SS interactions
   Ideal decouplers are merely gains:

\[
T_{12} = -\frac{K_{P12}}{K_{P11}} \quad (18-85)
\]

\[
T_{21} = -\frac{K_{P21}}{K_{P22}} \quad (18-86)
\]

3. **Nonlinear Decoupling**
   Appropriate for nonlinear processes.
Wood-Berry Distillation Column Model
(methanol-water separation)

Feed $F$

Reflux $R$

Distillate $D$, composition (wt. %) $X_D$

Steam $S$

Bottoms $B$, composition (wt. %) $X_B$
Wood-Berry Distillation Column Model

\[
\begin{bmatrix}
y_1(s) \\
y_2(s)
\end{bmatrix} = \begin{bmatrix}
\frac{12.8e^{-s}}{16.7s + 1} & \frac{-18.9e^{-3s}}{21s + 1} \\
\frac{6.6e^{-7s}}{10.9s + 1} & \frac{-19.4e^{-3s}}{14.4s + 1}
\end{bmatrix}\begin{bmatrix}
u_1(s) \\
u_2(s)
\end{bmatrix}
\]

(18–12)

where:

\( y_1 = x_D \) = distillate composition, \%MeOH

\( y_2 = x_B \) = bottoms composition, \%MeOH

\( u_1 = R \) = reflux flow rate, lb/min

\( u_1 = S \) = reflux flow rate, lb/min
Figure 19.13. An experimental application of decoupling (noninteracting) control to a distillation column [3].