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A finite element model of the stress field in a star-shaped inclusion

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Abstract

The elastic stress field in the neighborhood of an inclusion in the shape of a five-pointed star embedded in a homogeneous matrix subject to a remote uniaxial strain is determined using the finite element method in both plane stress and plane strain. The results do not support a recent preliminary analytical result that both the individual stress components and the effective stress distribution inside the inclusion should be uniform.

1. Introduction

A recent analytical calculation of the elastic stress field around an isotropic, elastic, star-shaped inclusion [1] has motivated the finite element calculations described in this note. The configuration to be analyzed is shown schematically in Fig. 1. The inclusion is a centro-symmetric, five-pointed star with apex angles of 36°, with Young's modulus and Poisson's ratio E_{star} and υ_{star}, respectively, while the matrix material will be denoted as E_{matr} and υ_{matr}. The boundary condition consists of remote uniaxial tension in either the x- or y-direction as indicated in the figure. The star is assumed to be perfectly bonded to the matrix and no restrictions are placed on the maximum stresses which can be sustained prior to failure or debonding. Both plane stress and plane strain analyses will be performed.

2. Model implementation

2.1. Finite element model

The FE mesh, consisting of a nonuniform distribution of six-noded, quadratic triangular elements, is partially shown in Fig. 2. The elements chosen can be used with both a plane strain or plane stress option. The outer boundary of the mesh measures 10 by 10 mm^2 and the inclusion has a diameter of approximately 2 mm. It is expected that edge effects, due to the finite dimensions of the mesh, could cause the final results to differ from an analytical solution for an inclusion in an infinite body, although it will be assumed that these effects will not significantly disturb the magnitude of the results. Additionally, calculations have been performed on the

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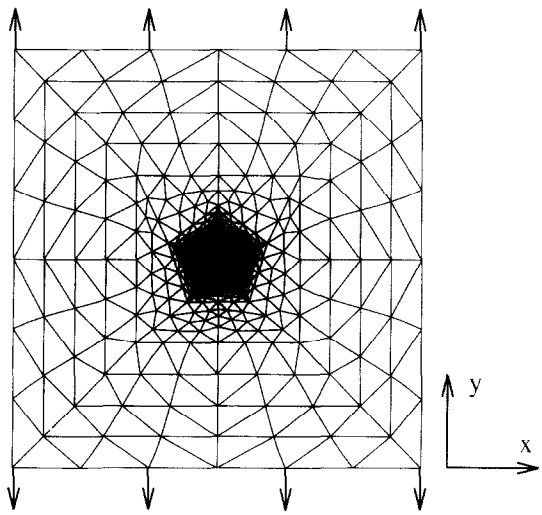

Fig. 1. Schematic of finite element mesh used to model a star-shaped inclusion showing remote displacement boundary conditions for the y-loading direction.

mesh in Fig. 2 assuming a homogeneous material ($E_{star} = E_{matr}$ and $\upsilon_{star} = \upsilon_{matr}$), and perturbation from a uniform stress field was found to be minimal in the central region. The boundary conditions are also indicated schematically in Fig. 1. Specifically, an upward displacement of 0.001 mm is imposed while the y-displacements along the lower edge nodes are fixed. All nodes are free to move in the x- and y- directions with the exception of nodes on the upper and lower edges which may only move freely in the x-direction. For numerical stability, a single edge node is fixed in the x–y plane in order to prevent rigid body motions. The calculations are repeated for tension in the x-direction as well.

Due to the arbitrary nature of the solution proposed by Mura [1], we take two specific sets of elastic constants: $E_{star} = 100$ GPa and $E_{matr} = 50$ GPa with $\upsilon_{star} = \upsilon_{matr} = 0.3$, as well as the reverse, $E_{star} = 50$ GPa and $E_{matr} = 100$ GPa. In terms of

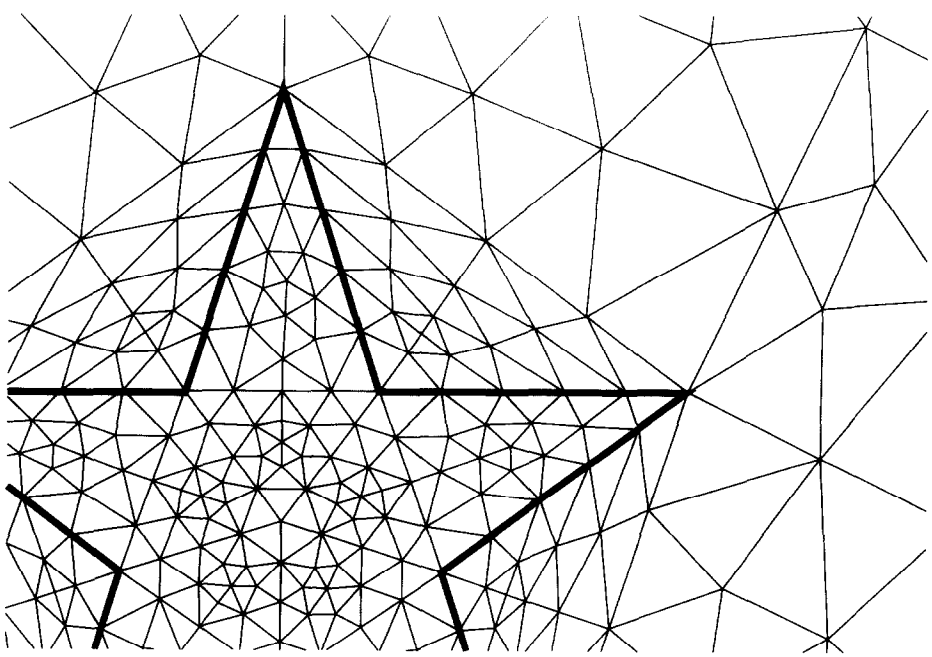

Fig. 2. Actual FE mesh magnified to show details near a star apex.

Dundurs' parameters (for precise definitions and discussion of significance, see Refs. [2] and [3]), the elastic constants correspond to $\alpha = \pm 1/3$ and $\beta = \pm 2/21$.

3. Results and discussion

Contour plots for the stress component σ_{yy} and the effective stress [defined as $\sqrt{(3s_{ij}s_{ij}/2)}$,

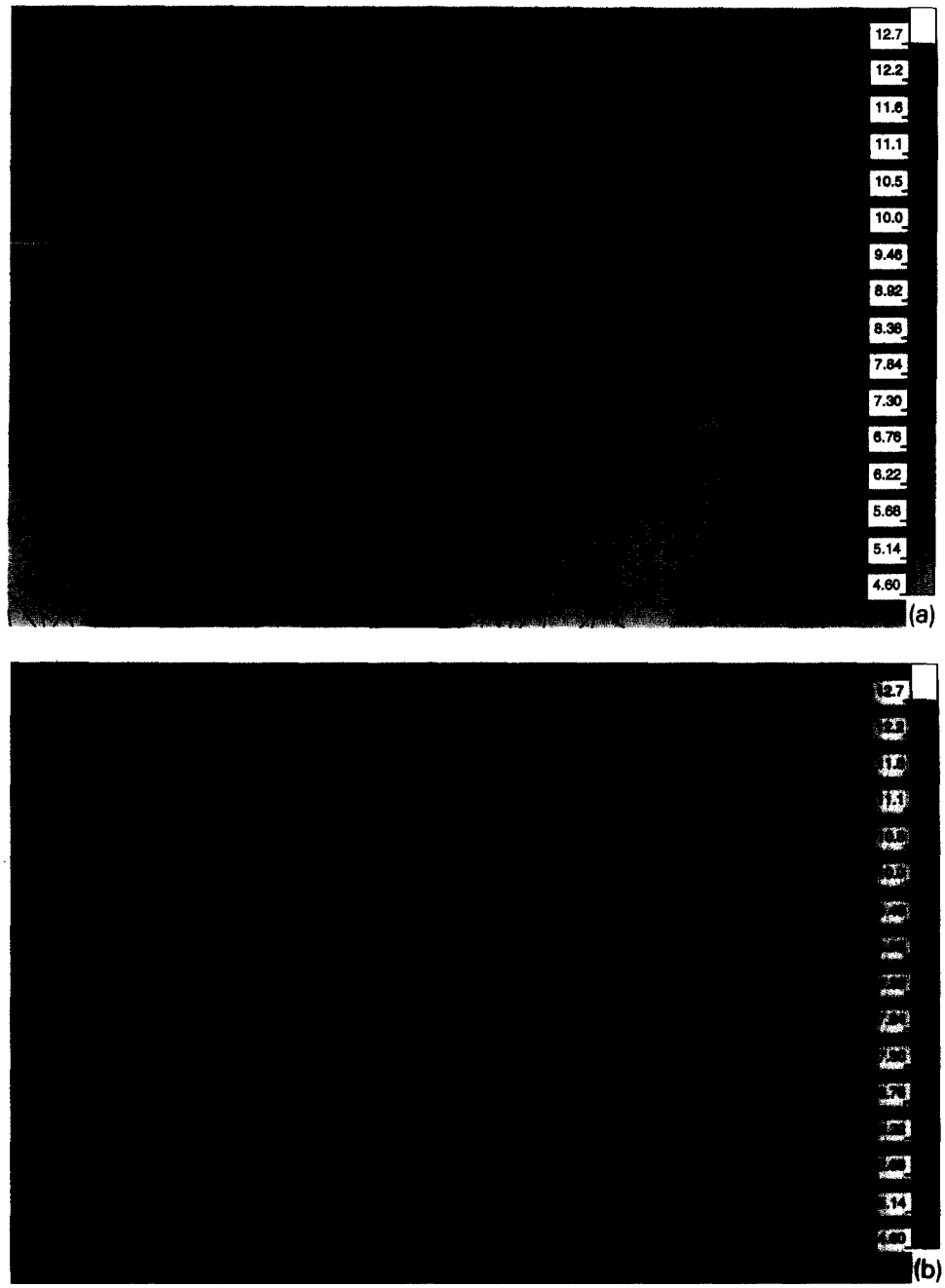

Fig. 3. (a) σ_{yy} distribution for stiffer inclusion for model loaded in the y-direction. (b) σ_{yy} distribution for more compliant inclusion.

where $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$] are shown in Figs. 3 through 6 for loading in the y- and x-directions, respectively. Analyzing these figures shows that stress components are far from homogeneous for both far-field displacement orientations. As expected from numerous analyses of stresses near

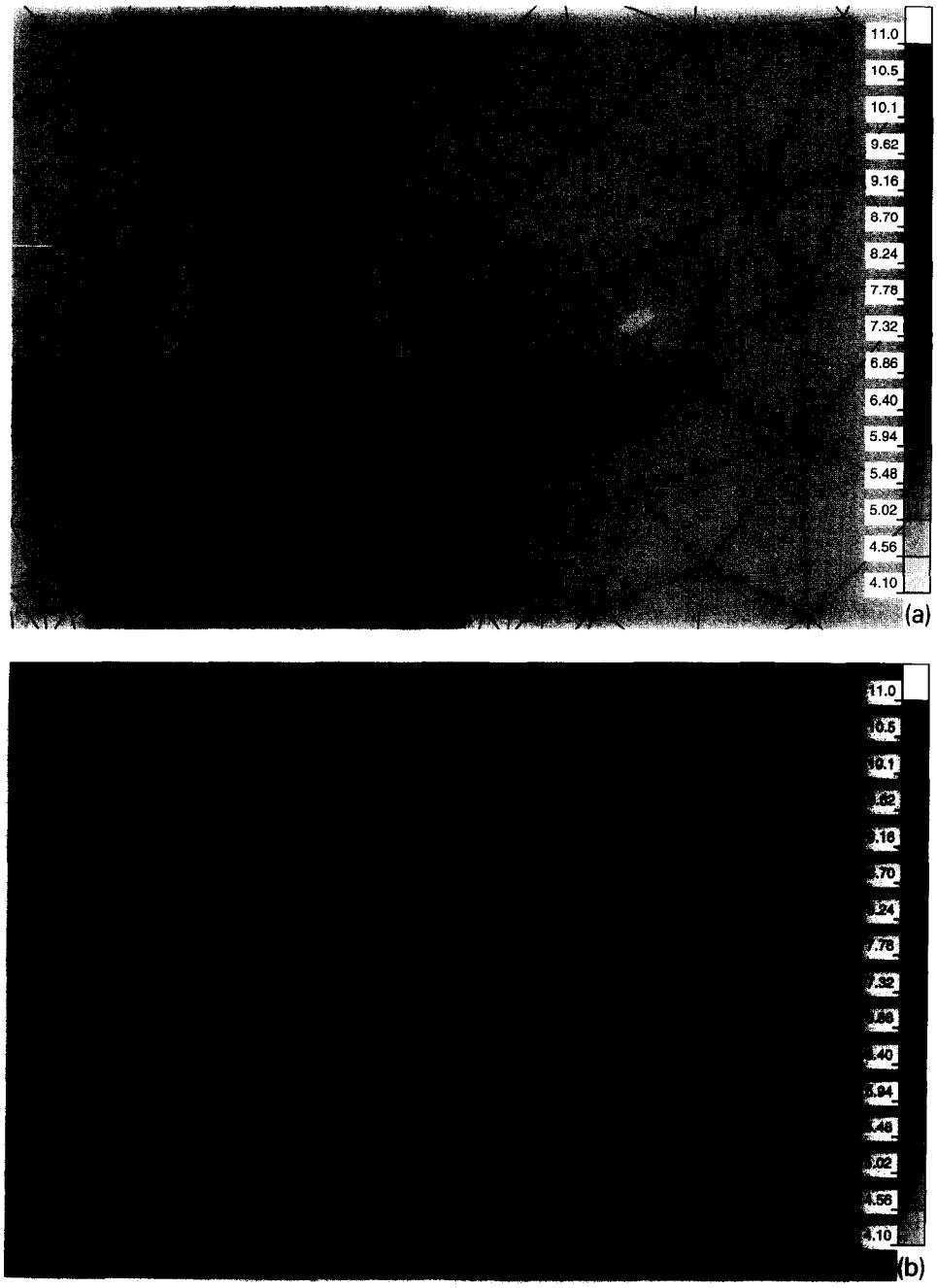

Fig. 4. (a) Effective stress distribution for stiffer inclusion. (b) Effective stress distribution for compliant inclusion loaded in y-direction.

wedge-shaped inclusions, the various stress components are essentially singular in the vicinity of the outer apexes, indicating that plasticity or cracking (depending on the type of material), which could lead to interfacial debonding, would be expected to initiate here.

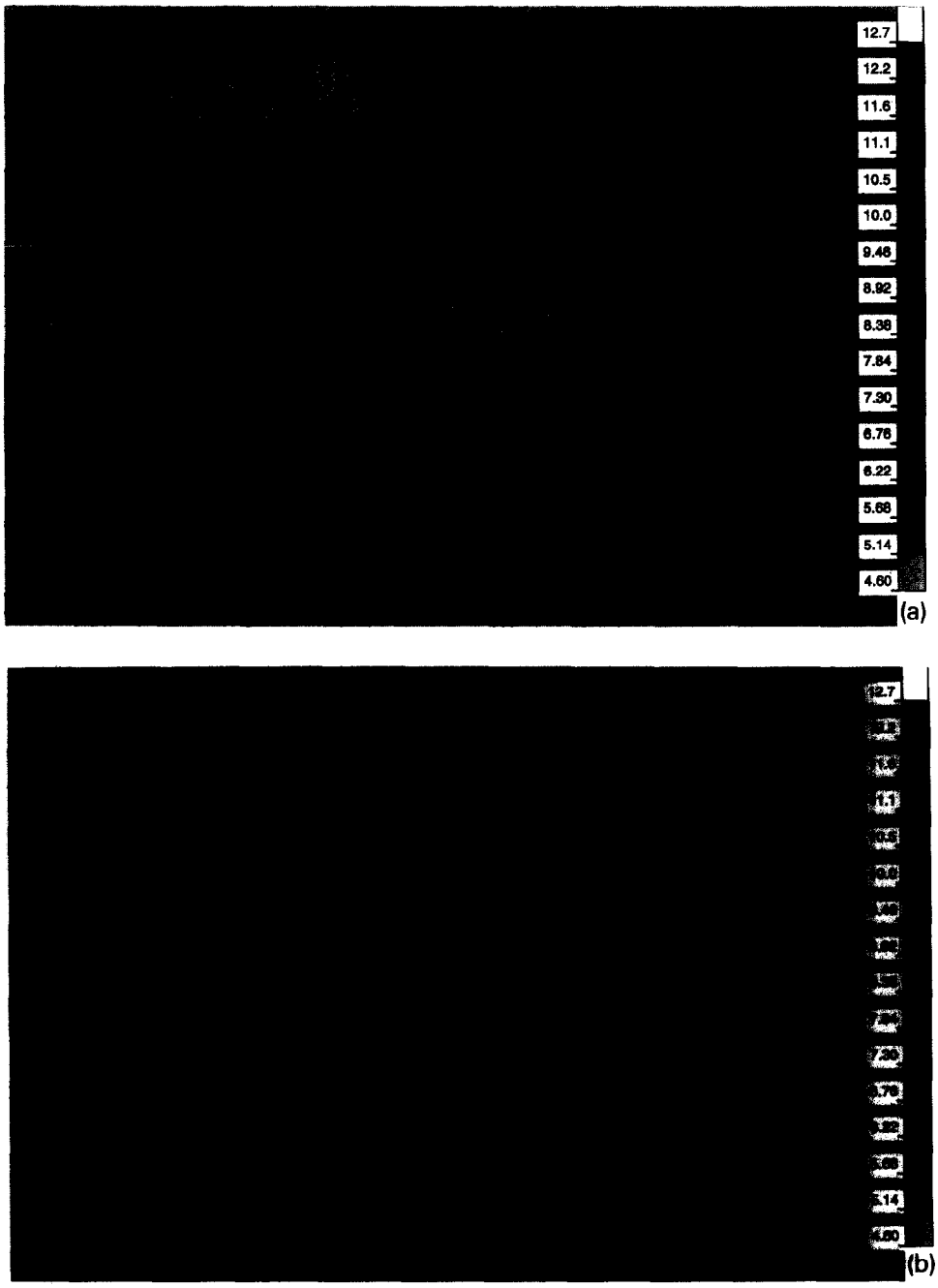

Fig. 5. σ_{xx} distribution as in Fig. 3. Loading in x-direction.

We note that the FEM results of the stress components within the inclusion deviate strongly from uniformity (more than a factor of 2), which is in direct contradiction to preliminary analytical modelling on star shaped inclusions in infinite elastic media [1]. While it is certainly possible

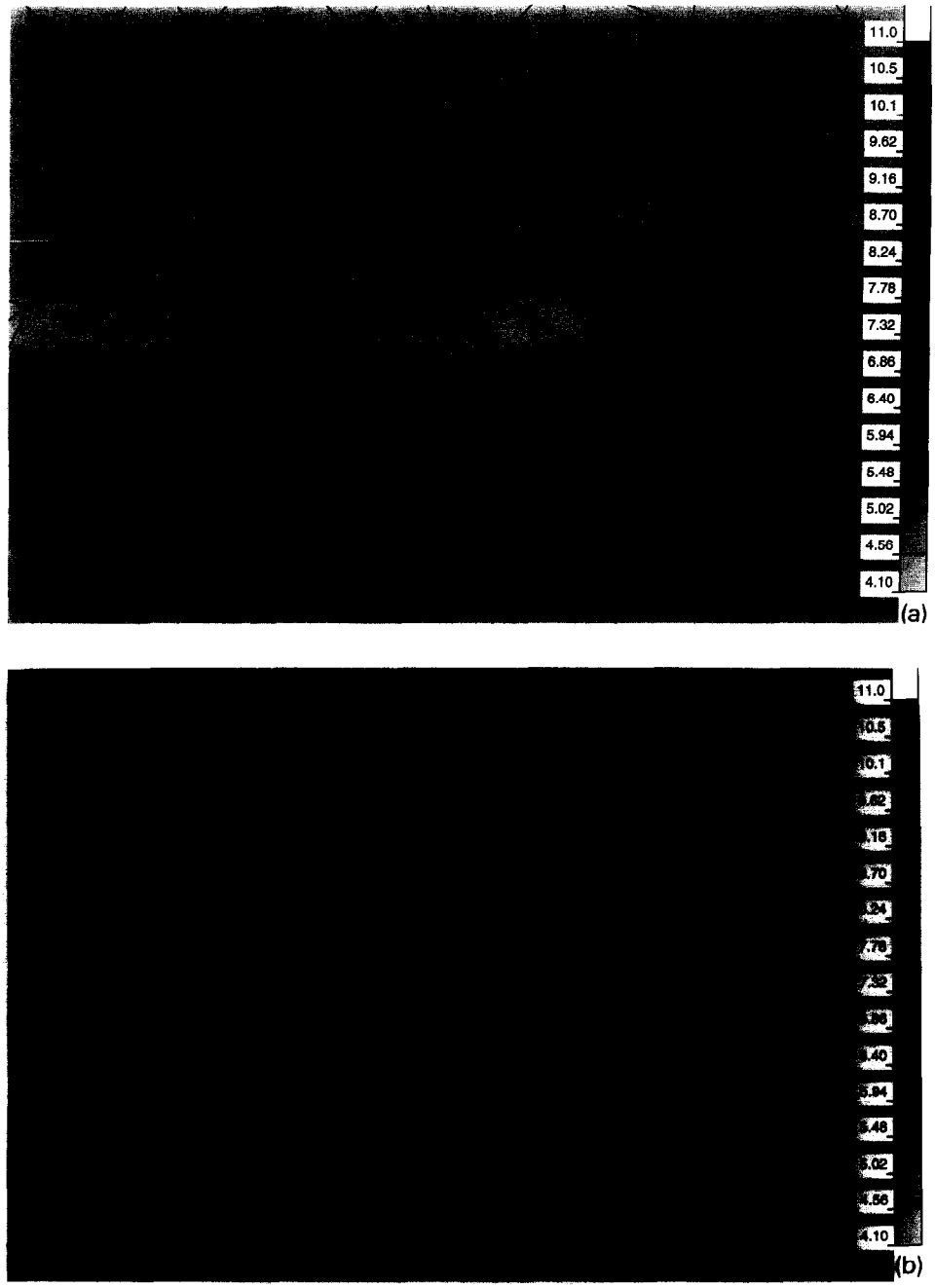

Fig. 6. σ_{xx} distribution as in Fig. 4. Loading in x-direction.

that the calculations shown here are not completely accurate due to mesh resolution or boundary interaction effects, it is not likely that these effects are responsible for the great disagreement between these and the analytical results.

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