Detection of Changes in Global Structural Stiffness Coefficients Using Acceleration Feedback

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Abstract: This technical note presents an extension of a previous study where two methods for detecting structural damage have been developed by using displacement and velocity measurements. In this study, acceleration feedback is used in detecting changes in global structural stiffness coefficients of lumped-mass-shear-beam models. The previously developed method relies on the decoupling of effects of changes in stiffness at different locations and the use of displacement or velocity feedback has proven to be effective. Extension to the use of acceleration feedback using existing formulation is not trivial in that the desired decoupling effect cannot be achieved by simple coordinate transformation because the acceleration itself is directly related to the stiffness coefficients. An approach to circumvent this difficulty is presented and it involves increasing the order of time derivatives of the linear system so that the acceleration becomes the "velocity" of the new system. The performance of the proposed method is demonstrated using an illustrative example of a three-story model with stiffness changes at different floors. Numerical studies are also conducted to evaluate the time horizons required to normalize monitor outputs for the effective and efficient detection of stiffness changes.

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Introduction

In a previous study by Ma et al. (2005), a method for simultaneously identifying and locating changes in structural stiffness has been developed. The measured structural responses were assumed to be displacements and velocities. However, these types of structural responses are not as easily and accurately obtainable as accelerations, which have been commonly obtained through the use of accelerometers. Accelerometers are both relatively inexpensive and reliable. Acceleration feedback has been used in the control of structures (Dyke et al. 1996; Mei et al. 2002; Christenson et al. 2003) and Spencer et al. (1995) have successfully implemented acceleration feedback strategies and have shown that they are effective, robust, and practically implementable.

In this study, the previously developed method based on displacement or velocity measurements for the detection of changes in global structural stiffness coefficients is extended to the case of acceleration feedback. This is not trivial due to the fact that the desired decoupling effect cannot be readily achieved by simple coordinate transformation because the acceleration itself is directly related to the stiffness coefficients. An approach to circumvent this difficulty is presented and it involves increasing the order of time derivatives of the linear system so that the acceleration becomes the "velocity" of the new system. The previously developed approach can thus be valid for acceleration measurements. The performance of the proposed method is demonstrated using an illustrative example of a three-story model with stiffness changes at different floors.

For all cases considered, monitor outputs are normalized in order to obtain a dimensionless indicator for stiffness changes. The effectiveness of the damage detection algorithm was found in Ma et al. (2005) to be dependent on the time horizon of past measurements used to normalize measured responses. Herein, the results of extensive numerical simulations are reported to study how the time horizon can be chosen small enough that the quick detection of stiffness changes is possible, but not so small that inaccuracies are misdiagnosed as stiffness changes. This study suggests that using acceleration feedback may require a longer time horizon to accurately start monitoring for stiffness changes possibly due to the reduced accuracy from using the time derivative of acceleration, which is an order of magnitude less accurate than the original measurement. Such possible accuracy loss due to performing derivatives can be remedied and the time horizon can be shortened with the use of smaller time steps in performing the integration over the selected time horizon, which is limited, however, by the time steps chosen when collecting the earthquake data. On the other hand, as long as one can clearly distinguish the monitor output data or curves that indicate stiffness changes, the purpose of identifying the stiffness changes can be fulfilled.

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Algorithm Formulation

The problem of detecting changes in the stiffness coefficients of linear lumped-mass-shear-beam building structures subjected to seismic excitation is considered. The governing equation of such a structure can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}_s\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\cdot\mathbf{1}w \tag{1}$$

where $\mathbf{x}$=displacement vector; w denotes the ground acceleration time history of the earthquake; $\mathbf{M}$, $\mathbf{K}$, and $\mathbf{C}_s$=mass, stiffness, and damping matrices of the original structure, respectively; and $\mathbf{1}$ denotes a column vector with all elements being 1.

A solution based on the measured state vectors, i.e., $\mathbf{x}$ and $\dot{\mathbf{x}}$, has been developed by designing a "monitor" for every stiffness coefficient (Ma et al. 2005). One monitor responds only to changes in the corresponding stiffness coefficient; i.e., its output will not be affected by changes in other stiffness coefficients; thus, the occurrence and location of stiffness changes can be determined. A monitor is essentially a linear observer, which has the following state space representation:

$$\dot{\tilde{x}}_{a,i} = \tilde{A}_a\tilde{x}_{a,i} + G_i w - E_{a,i}\mathbf{y}, \quad i = 1,2,\ldots,n$$

$$\tilde{z}_{a,i} = C_{ra}\tilde{x}_{a,i} = C_{y,i}\mathbf{y} \tag{2}$$

where

$$\tilde{A}_a = \begin{bmatrix} 0 & \dfrac{\sum m_i/n}{m_i} \\ -\dfrac{k_i}{\sum m_i/n} & -\gamma \end{bmatrix},$$

$$G_i = \frac{1}{\sum_{i=1}^{n} m_i/n}\begin{bmatrix} 0 & \sum_{j=1}^{n} m_j \end{bmatrix}^r, \quad C_{ra} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$C_{y,i} = \frac{1}{\sum m_i/n}\begin{bmatrix} 0_{i\times(i-1)} & m_i & \cdots & m_n \end{bmatrix} \tag{3}$$

$$E_{a,i} = \begin{bmatrix} 0_{i\times(i-2)} & -1 & 0 & \dfrac{m_{i+1}}{m_i} & \cdots & \dfrac{m_n}{m_i} \\ 0_{i\times(i-2)} & \dfrac{-c_i}{\sum m_i/n} & \dfrac{c_i-\gamma m_i}{\sum m_i/n} & \dfrac{-\gamma m_{i+1}}{\sum m_i/n} & \cdots & \dfrac{-\gamma m_n}{\sum m_i/n} \end{bmatrix} \tag{4}$$

in which m_i=mass of the ith floor; n=number of floors; and $\mathbf{y}$=velocity measurements. The error of the observer $\tilde{z}_{a,i}$ is sufficiently small, i.e., $r_i = \tilde{z}_{a,i} - C_{y,i}\mathbf{y} \approx 0$ if the ith stiffness coefficient does not change, i.e., $\Delta k_i = 0$ and only in the case where $\Delta k_i \neq 0$, $r_i \neq 0$. Thus, by monitoring such error, the changes in stiffness coefficients can be detected.

In practice, absolute accelerations can be measured much more easily as compared to velocities. The absolute acceleration measurements have the form

$$\ddot{\mathbf{x}} + \mathbf{1}w = -\mathbf{M}^{-1}(\mathbf{C}_s\dot{\mathbf{x}} + \mathbf{K}\mathbf{x}) \tag{5}$$

Since the measured absolute acceleration contains information about the system stiffness, designing a monitor that responds only to one stiffness coefficient is not directly possible using the approach developed in the previous study. An approach of circumventing such a difficulty is proposed based on increasing the order of the system so that accelerations are the states of the system; furthermore, the accelerations become pseudovelocities of the system with an augmented order.

Differentiating Eq. (5) with respect to time yields

$$\mathbf{M}\dddot{\mathbf{x}} + \mathbf{C}_s\ddot{\mathbf{x}} + \mathbf{K}\dot{\mathbf{x}} = -\mathbf{M}\cdot\mathbf{1}\dot{w} \tag{6}$$

If it is defined that

$$\dot{\tilde{\mathbf{x}}}_1 = \ddot{\mathbf{x}} = \tilde{\mathbf{x}}_2 \tag{7}$$

then

$$\dot{\tilde{\mathbf{x}}}_2 = \dddot{\mathbf{x}} = -\mathbf{M}^{-1}(\mathbf{K}\dot{\mathbf{x}} + \mathbf{C}_s\ddot{\mathbf{x}}) - \mathbf{1}w \tag{8}$$

Note that $\tilde{\mathbf{x}}_2$=floor accelerations relative to the ground.

The following equations are thus readily established:

$$\begin{bmatrix} \dot{\tilde{\mathbf{x}}}_1 \\ \dot{\tilde{\mathbf{x}}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}_s \end{bmatrix}\begin{bmatrix} \tilde{\mathbf{x}}_1 \\ \dot{\tilde{\mathbf{x}}}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ -\mathbf{1} \end{bmatrix}\dot{w}$$

$$\mathbf{y} = \tilde{\mathbf{x}}_2 + \mathbf{1}w \tag{9}$$

In Eq. (9), the accelerations relative to the ground are states ("velocities") of the new system and they can be obtained by subtracting the ground acceleration from the measured absolute acceleration. Thus, a monitor can be designed for every stiffness coefficient for system (9) using the approach summarized in Eqs. (2)–(4), which is the same as the one based on the velocity feedback (Ma et al. 2005).

As shown in Eq. (9), an obvious difficulty encountered using the acceleration feedback is that the time derivatives of the ground acceleration due to external earthquake excitations are needed in the monitor design process. Such a derivative is approximated in this study by using the basic linear central finite difference technique. The sampling time interval, Δt, is chosen based on the characteristic of the measured ground acceleration due to the earthquake excitation, w. This Δt must be sufficiently small to accurately represent the response curves, especially for those half-cycles with high peaks and short periods. The derivative obtained this way is linear for any time instance between two measurement points. If the sampling data are not sufficiently refined and the oscillation during the small Δt time period (between two sampling points) is noticeable, the approximation would involve error to some degree. This would result in having monitors with a low sensitivity.

For the examples illustrated in this study, the *N-S* component of the 1940 El Centro earthquake was chosen as the external excitation with a sampling frequency of 50 Hz, i.e., sampling time interval of 0.02 s. The dominant frequency of this excitation was found to be 1.9 Hz with a period of about 0.5 s, which is significantly lower than the sampling frequency of 50 Hz and significantly higher than the sampling time interval of 0.02 s chosen. The derivative obtained by the presently proposed finite difference method is evaluated by a series of subsequent numerical examples.

Monitor outputs are normalized with respect to the measurements as follows:

$$r_{\text{norm},i}(t) = \sqrt{\frac{\int_{t-t_h}^{t} r_i^2(\tau)d\tau}{\int_{t-t_h}^{t} \bar{y}_i^2(\tau)d\tau}}, \quad \bar{y}_i(t) = C_{y,i}\mathbf{y} \qquad (10)$$

where $r_{\text{norm},i}$=normalized output for the ith monitor; r_i=output for the ith monitor; $\mathbf{y}$=measurement vector; and t_h=integration time horizon of past measurements used for normalization. Such normalized output is dimensionless and thus serves as a better indicator for stiffness changes.

Extensive numerical simulations are conducted here to study the integration time horizon t_h and time intervals Δt required to integrate through time history responses in order to normalize the monitor outputs and to design a monitor that would not only quickly begin monitoring for stiffness changes but also detect such changes accurately, i.e., without false readings. The selection of t_h is crucial and it is studied numerically and discussed in detail through illustrative examples in the following section.

Numerical Simulations

The structure considered in this study was a three-story model used by Yang et al. (1995). The mass, stiffness, and the damping coefficients of each floor were assumed as 1,000 kg, 980 kN/m, and 1.407 kN-s/m, respectively. The *N-S* component of the ground acceleration of the 1940 El Centro earthquake was used as excitation. The percent stiffness changes are indicated by α_i, where "i"=floor number. For the original structure, α_i=0%. In this study, no sensor noise was added to the measurements in order to focus on evaluating the performance of the proposed algorithm in the ideal case. The effect of the sensor noise needs further in-depth investigation.

Simulation results for different stiffness change scenarios of this model are shown in Fig. 1 for the acceleration feedback. It can be observed that in each case when there is a stiffness change ($\alpha_i > 0$), the resulting monitor output deviates from zero. Normalized monitor outputs also indicate the presence of damage by a clearly increased value as compared to the unchanged case. It was also noted that monitor output curves for floors with stiffness changes, in addition to having a clear jump as compared to the monitor curves for the floors without stiffness changes, become more flat as the value of t_h is increased, as can be seen in Fig. 2 (α_2=20%) and Fig. 3 (α_2=20%, α_3=10%).

By increasing t_h, it was observed that the normalized monitor output, $r_{\text{norm},i}$, converges to a specific value for each scenario analyzed. The results are summarized in Table 1. From these results, it can be noticed that for each case when there is a stiffness change, the monitor outputs for the corresponding floor are clearly increased as compared to the monitor outputs of the floors with unchanged stiffness. Also, the ratio between the monitor outputs for the floors with stiffness changes and those without, which is indicated in Table 1 by r^*, could be used as an indicator of detected damage. Furthermore, the converged values of the normalized monitor output seem to be the same as the mean values over the entire time history for each example considered. For illustrative purposes, error bars for the normalized output obtained using different integration time horizons are shown in Fig. 4 for the case when the second and third floors were assumed damaged. Each error bar shows the maximum and minimal values and the mean value of the corresponding normalized output,

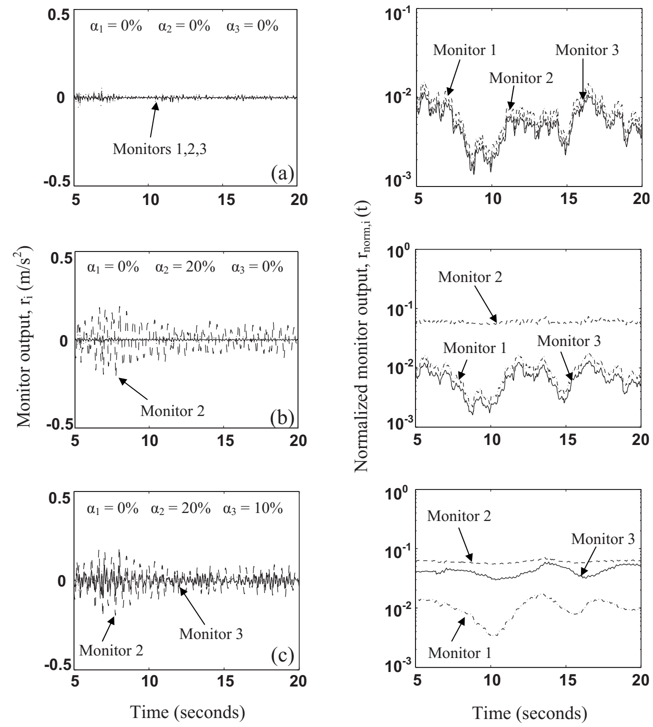

Fig. 1. Damage detection for the three-story model using acceleration feedback: (a) no damage (all α_i=0%, t_h=0.5 s); (b) second floor is damaged (α_2=20%, t_h=0.5 s); and (c) second and third floors are damaged (α_2=20% and α_3=10%, t_h=2 s)

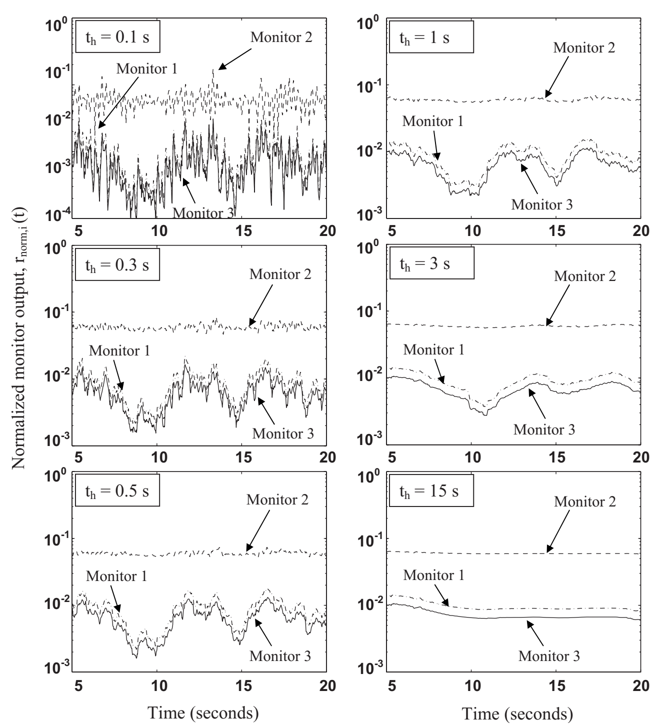

Fig. 2. Effect of the integration time t_h on monitor performance for the three-story model using acceleration feedback: Case (b) second floor is damaged (α_2=20%)

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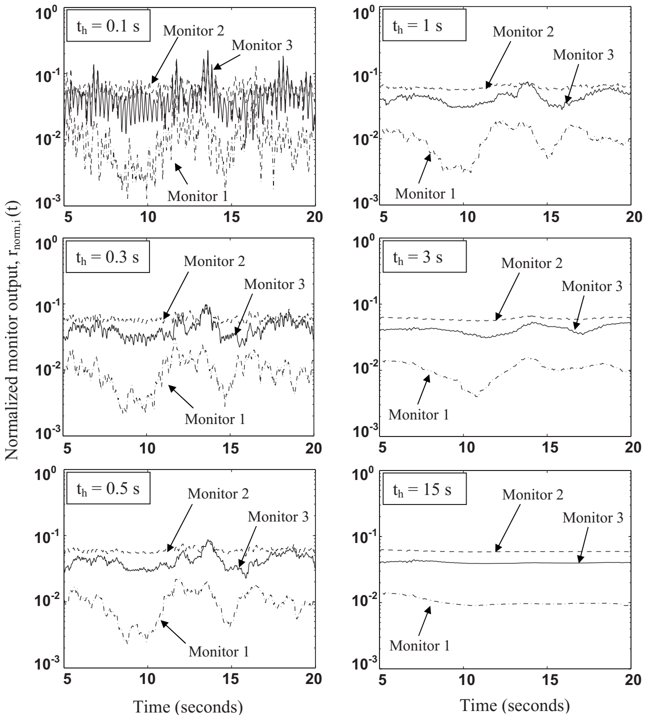

Fig. 3. Effect of the integration time t_h on monitor performance for the three-story model using acceleration feedback: Case (c) second and third floors are damaged ($\alpha_2=20\%$ and $\alpha_3=10\%$)

which is marked by a circle, square, and triangle for Monitors 1, 2, and 3, respectively. It is seen that the integration time horizon does not affect the mean value of the normalized output noticeably. However, a shorter integration time horizon generates larger fluctuations of the normalized output as demonstrated by the overlaps of the error bars, which may increase the possibility of false warnings. Particularly for the cases considered, the integration time horizon t_h required to design an accurate monitor was found to be 0.3 s for the case of single stiffness change (as shown in Fig. 2) while it was necessary to select $t_h=0.5$ or even 1 s if there were changes in multiple floors, as shown in Fig. 3.

Concluding Remarks

A method for detecting changes in stiffness coefficients using acceleration measurements has been presented. The method is

Table 1. Normalized Monitor Output, $r_{\text{norm},i}(t)$, for the Three-Story Model; Full Time History

Floors with stiffness changes	Normalized monitor output, $r_{\text{norm},i}$	Acceleration feedback
Second ($\alpha_2=20\%$)	r_1	0.0095
	r_2 (r^*)	0.062 (6.5)
	r_3	0.0095
Second, third ($\alpha_2=20\%$, $\alpha_3=10\%$)	r_1	0.0095
	r_2, (r^*)	0.062 (6.5)
	r_3, (r^*)	0.04 (4)

Note: r^* indicates the ratio between the monitor output for damaged and undamaged floors, i.e., $r^*=r_{\text{norm},i}(\text{damaged floor})/r_{\text{norm},i}(\text{undamaged floor})$.

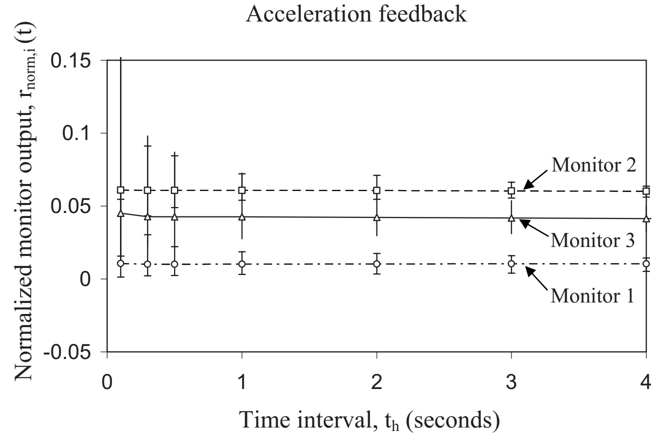

Fig. 4. Normalized monitor output for the three-story model: Case (c) second and third floors are damaged ($\alpha_2=20\%$ and $\alpha_3=10\%$)

based on increasing the order of time derivatives of the linear system so that the stiffness changes can be decoupled similar to the case presented in a previous study where displacement and velocity measurements were used. In this study, it was shown that stiffness changes could be detected for the example considered using the proposed method. In the cases studied, the separation of the normalized outputs of the monitors corresponding to the floors with stiffness changes and those for the ones without changes were obvious. It is noted that the detection algorithm is based on decoupling the effects of changes in system parameters on the structural response. For lumped-mass models, which could be those of buildings and bridges, the desired decoupling effect may be relatively easy to obtain. For more complex models, decoupling may be difficult, but still possible if the corresponding response can be determined.

It was also shown that the integration time horizon that is used for normalization plays an important role in detecting stiffness changes. A longer time horizon would delay the detection, but have better accuracy while a shorter one would enable faster detection, but the possibility of false warnings may also increase. Due to the loss of numerical accuracy resulting from the use of the time derivative of acceleration, monitors designed using acceleration feedback may require a longer integration time horizon to accurately start monitoring for damage. Such accuracy loss due to performing derivatives can be remedied and the required integration time horizon can be shortened with the use of smaller time steps in performing the integration, which is limited by the time steps chosen for collecting the earthquake data.

It is noted that in order to focus on the concept of using acceleration measurements to detect stiffness changes, some practical issues, such as the measurement noise, relation of structural damage (missing members, cracks, etc.) to changes in the interstory stiffness, effect of different earthquakes and/or structural types on the integration time horizon t_h, real-time damage detection, etc., require future in-depth study. As a logical next step, these issues, in combination with detailed theoretical and experimental investigations, are thus recommended in a future study.

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