

Letters

Performance Study of an Adaptive Controller in the Presence of Uncertainty

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Abstract—This letter addresses the performance study of an adaptive controller in the presence of gain and delay uncertainties. The nonlinearities in the controller presented here do not allow an analytical study of the effect of uncertainties on performance. A numerical study is done instead using the sampling, learning, prediction, and optimization capabilities of the software Global Optimization, Sensitivity and Uncertainty in Models (GoSUM).

Index Terms—Delay systems, nonlinear control systems, robustness, stability.

I. INTRODUCTION

Adaptive control can help deliver both stability and good response for systems with variability in parameters that can either be predicted or are uncertain. The goal of adaptive control is to adjust to unknown or changing plant parameters. This is accomplished by either changing parameters in the controller to minimize error, or using plant parameter estimates to change the control signal. However, it has been shown that the introduction of small disturbances into an adaptively controlled system could deteriorate the efficiency of the control and lead to instability of the closed-loop system because of unmodeled dynamics [1]–[3]. In recent years, new adaptive control methodologies have addressed this potential instability problem but are applicable only in somewhat restrictive situations [4]–[6]. Another limitation to adaptive controller is that, in practice, systems with uncertain time delays are frequently encountered. Time-delay effects on robust stability have been discussed in details for linear systems [7]; however, few methods have been designed to deal with systems subject to time delays with unknown parameters or unknown nonlinear functions [8]. The analysis and quantification of the performance and stability of adaptively controlled systems is needed to use a practical control system design with confidence. Because of their inherent complexity, adaptive controllers can be difficult to analyze or optimize. In the presence of nonlinearity, there exist few analysis tools. Verification tests are currently developed using collected data [9] or probabilistic Monte Carlo methods [10]. New adaptive controller strategies with greater robustness have been developed for aircraft control [11], [12]. In this letter, we analyze the stability and robustness of an adaptive control design with and

without gains and time delays applied to a nonlinear-in-control fighter aircraft described in [13] flying at or near stall angle of attack in the presence of aerodynamic uncertainties. For analysis we use the Global Optimization, Sensitivity and Uncertainty in Models (GoSUM) software [14].

GoSUM is a tool for global sensitivity analysis, optimization, uncertainty analysis, and model reduction. It employs kernel-based algorithms to accurately learn various models either from data samples or from a model executable. It generates samples that are much more uniformly distributed than Monte Carlo samples, thus resulting in a fewer number of samples needed to achieve output statistics. Once a model is learned, using support vector regression [15] for continuous outputs or support vector machine (SVM) for binary outputs, GoSUM can perform the following tasks: predict model outputs for a given set of input factors; identify input factors responsible for model variability; quantify global uncertainty in model outputs; and optimize an objective function in the presence of equality and inequality constraints. All analysis types in GoSUM are designed to handle tens of thousands of effects on multiple outputs of interest in a short time, permitting more rapid examination of the effect of uncertainties on the model than the current verification tests.

The structure of this letter is as follows. Section II describes the fighter aircraft model and the adaptive controller. Four additional uncertain parameters, two gains and two time delays, are added to the system. The measures for the control effort and the errors related to tracking and to adaptation are also presented. Section III presents the resulting tracking and adaptation results. Section IV discusses the controller performance and robustness in presence of gain and time delays.

II. IMPLEMENTATION DETAILS

A. Fighter Aircraft Model

The model for aircraft pitch dynamics that we use as an example in this letter is described in full detail in [13] and is given by

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} = A_p \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + b_p (\delta_e(t) + f(\alpha(t), \delta_e(t))) \quad (1)$$

where α is the angle of attack, q is the pitch rate, δ_e is the elevator deflection, and $f(\alpha(t), \delta_e(t))$ represents the unknown nonlinear-in-control and the unknown nonlinear-in-state α effects. The control objective is to design the elevator deflection $\delta_e(t)$ such that the angle of attack $\alpha(t)$ tracks a bounded time-varying reference signal $r(t)$. The nominal control elevator deflection $\delta_e(t) = \delta_{e_{\text{nom}}}(t)$ is nominal in the absence of uncertainties $f(\alpha(t), \delta_e(t)) = 0$ and is designed with a proportional integral (PI) linear quadratic regulator (LQR) controller to achieve the tracking objective: i.e., $\alpha(t) \rightarrow r(t)$ as $t \rightarrow \infty$, if $r(t)$ is a constant.

The plant/PI controller system is given by a set of three nonlinear ordinary differential equations (ODEs) as

$$\begin{cases} \dot{\alpha}_I = \alpha(t) - r(t), \\ \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = A_p \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + b_p (\delta_e(t) + f(\alpha(t), \delta_e(t))) \end{cases} \quad (2)$$

with the baseline control signal given by

$$\delta_e(t) = -k_{LQR}^T \begin{bmatrix} \alpha_I(t) \\ \alpha(t) \\ q(t) \end{bmatrix} =: \delta_{e_{\text{nom}}}(t). \quad (3)$$

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B. Adaptive Controller

To account for uncertain control-dependent nonlinearities in aggressive flight regimes, $f(\alpha(t), \delta_e(t))$, which is an adaptive augmentation of the linear optimal (LQR) controller, is used. $\delta_e(t) = \delta_{e_{\text{nom}}}(t) - \delta_{e_{\text{ad}}}(t)$. The unknown function $f(\alpha(t), \delta_e(t))$ is estimated using a radial basis functions (RBFs) method [16]. A set of RBFs $\phi : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is defined by $\phi(\alpha, \delta_e) = \phi_1(\alpha) + \phi_2(\alpha, \delta_e) := \theta^T \Phi_1(\alpha) + w^T \Phi_2(\alpha, \delta_e)$ where $\Phi_1(\alpha)$ is the vector of Gaussians independent of the controller δ_e , while $\Phi_2(\alpha, \delta_e)$ is the vector of the integrals of the Gaussians dependent upon δ_e . The vector $W^T = [\theta^T w^T]^T$ is the unknown that the adaptive controller will estimate. To implement the adaptive controller, the state predictor is designed with 12 RBFs. The fixed center of the four $\phi_1(\alpha)$ -type Gaussians are evenly distributed over $[-45 \text{ deg}, 45 \text{ deg}]$ and the fixed center of the eight $\phi_2(\alpha, \delta_e)$ -type basis functions are evenly distributed over $[-45 \text{ deg}, 45 \text{ deg}] \times [-30 \text{ deg}, 30 \text{ deg}]$. We choose to use the same fixed width for all RBFs: $\delta = \rho = 30$.

A state predictor can be constructed as

$$\begin{bmatrix} \dot{\hat{\alpha}}_I(t) \\ \dot{\hat{\alpha}}(t) \\ \dot{\hat{q}}(t) \end{bmatrix} = A_r \begin{bmatrix} \hat{\alpha}_I(t) \\ \hat{\alpha}(t) \\ \hat{q}(t) \end{bmatrix} + \begin{bmatrix} -r(t) \\ 0 \\ 0 \end{bmatrix} + b \left(-\delta_{e_{\text{ad}}}(t) + \hat{W}(t)\Phi(\alpha(t), \delta_e(t)) \right) \quad (4)$$

where $A_r = A_p - b_p k_{LQR}$ and $\hat{W}(t)$ is the estimate of the unknown vector W and is given by the adaptive law [16]

$$\dot{\hat{W}}(t) = \Gamma \text{Proj}(\hat{W}(t), -\Phi(\alpha(t), \delta_e(t)) P_0 b) \quad (5)$$

where Γ is a positive-definite matrix of adaptation rates, $\text{Proj}(\cdot, \cdot)$ denotes the projection operator, P_0 is the solution to the Lyapunov equation $A_r P_0 + P_0 A_r = -Q_0$ for a positive-definite matrix Q_0 , and $e_s(t)$ is the prediction error defined as

$$e_s(t) = \begin{bmatrix} \hat{\alpha}_I(t) - \alpha_I(t) \\ \hat{\alpha}(t) - \alpha(t) \\ \hat{q}(t) - q(t) \end{bmatrix}. \quad (6)$$

We also introduce gains k_1 and k_2 as well as time-delays τ_1 and τ_2 in the control signal and output signal in the system. Those represent additional tuning parameters or uncertain parameters. The adaptive controller $\delta_{e_{\text{ad}}}$ is solved by using fast dynamics approximation. The set of ODEs becomes

$$\begin{cases} \dot{\alpha}_I = k_2 \alpha(t - \tau_2) - r(t) \\ \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = A_p \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + b_p (k_1 \delta_e(t - \tau_1) - f(\alpha(t), k_1 \delta_e(t - \tau_1))) \\ \begin{bmatrix} \dot{\hat{\alpha}}_I \\ \dot{\hat{\alpha}} \\ \dot{\hat{q}} \end{bmatrix} = A_r \begin{bmatrix} \hat{\alpha}_I \\ \hat{\alpha} \\ \hat{q} \end{bmatrix} + \begin{bmatrix} -r(t) \\ 0 \\ 0 \end{bmatrix} + b(-\delta_{e_{\text{ad}}}(t) + \hat{W}(t)\Phi(k_2 \alpha(t - \tau_2), \delta_e(t))) \\ \dot{\hat{W}}(t) = \Gamma \text{Proj}(\hat{W}(t), -\Phi(k_2 \alpha(t - \tau_2), \delta_e(t)) P_0 b) \\ \dot{\delta}_{e_{\text{ad}}} = -\frac{10}{3}(\delta_{e_{\text{ad}}}(t) - W(t)\Phi(k_2 \alpha(t - \tau_2), \delta_e(t))). \end{cases} \quad (7)$$

C. Controller Error Definitions

Both adaptive and nonadaptive controllers are implemented in a MATLAB code where the set of nonlinear delayed ODEs is solved using `dde23`. To characterize the performance of the controller, the code also calculates four error measures. The main goal of this

controller is to track a bounded and possibly time-varying reference signal. The error related to the tracking is defined as

$$E_{tr}(k_1, k_2, \tau_1, \tau_2) = \sqrt{\frac{1}{T} \int_0^T (\alpha(t) - r(t))^2 dt}. \quad (8)$$

The goal of the adaptive controller is to dominate the nonlinearity by $\delta_{e_{\text{ad}}}$ to keep the control effort small even in the presence of nonlinear perturbations. We define the PI control effort as $C_e(t) = b_p \delta_{e_{\text{nom}}}(t)$ in degrees per second with $\delta_{e_{\text{nom}}}(t)$ defined in (3). The adaptation error is computed by

$$E_{ad}(k_1, k_2, \tau_1, \tau_2) = \sqrt{\frac{1}{T} \int_0^T (f(\alpha, \delta_e) - \delta_{e_{\text{ad}}}(t))^2 dt}. \quad (9)$$

We characterize the control effort using the following two measures:

$$E_{C_e}(k_1, k_2, \tau_1, \tau_2) = \sqrt{\frac{1}{T} \int_0^T C_e(t)^2 dt} \\ E_{\max C_e}(k_1, k_2, \tau_1, \tau_2) = \max_{t \in [0, T]} |C_e(t)|. \quad (10)$$

III. TYPICAL TRACKING AND ADAPTATION

The simulation example in this letter follows the one in [13], i.e., an adaptive design for F-16 short-period dynamics calculated at the trimmed airspeed of 502 ft/s and angle of attack $\alpha = 2.11 \text{ deg}$. The controller can be tested with an approximated $f(\alpha(t), \delta_e(t))$ using some partial knowledge of the aerodynamic stability and control derivatives available from wind-tunnel experiments and/or theoretical predictions. For example

$$f(\alpha(t), \delta_e(t)) = \left((1 - C_0) e^{-\frac{(\alpha - \alpha_0)^2}{2\sigma^2}} + C_0 \right) \times (\tanh(\delta_e + h) + \tanh(\delta_e - h) + 0.01 \delta_e) \quad (11)$$

where $\alpha_0 = 2.11 \text{ deg} = 0.0368 \text{ rad}$, $\sigma = 0.25 \text{ rad}$, $C_0 = 0.1$, and $h = 0.14$.

The reference input of interest to track is

$$r(t) = \frac{95}{100} \left(\frac{1/2}{1 + e^{t-8}} + \frac{1}{1 + e^{t-20}} - e^{-0.2t} - \frac{1}{2} \right). \quad (12)$$

The Riccati equation is solved with the weighting matrices

$$Q = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R = 0.01 \quad \text{or} \quad R = 0.05. \quad (13)$$

We perform simulation tests using $R = 0.01$ and $R = 0.05$ for the LQR controller and compare the controller performances in six cases: $R = 0.01$, no nonlinear uncertainty, no adaptation; $R = 0.01$, nonlinear uncertainty, no adaptation; $R = 0.01$, nonlinear uncertainty, adaptation; $R = 0.05$, no nonlinear uncertainty, no adaptation; $R = 0.05$, nonlinear uncertainty, no adaptation; $R = 0.05$, nonlinear uncertainty, adaptation. The plots in Fig. 1(a) and (b) shows the tracking of the reference input signal, $r(t)$. We observe that better tracking is performed with $R = 0.01$ than with $R = 0.05$. This is confirmed by the calculation of $E_{st}(1, 1, 0, 0)$ given in Table I. It can also be seen that the uncertainty $f(\alpha, \delta_e)$ deteriorates the tracking, but in the case using the adaptive law the tracking error is very close to the tracking error without uncertainties. The plots in Fig. 1(c) and (d) show the performance of the adaptive law. In both cases ($R = 0.01$ and $R = 0.05$), $\delta_{e_{\text{ad}}}(t)$ and $\hat{W}(t)$ give good adaptive approximation of $f(\alpha, \delta_e)$ with better performance with $R = 0.05$ (see the value of $E_{ad}(1, 1, 0, 0)$ in Table I). We also observe that the system has difficulty adapting to the stiff change in the nonlinearity at $t = 1$. With $R = 0.01$, this change is underestimated whereas with

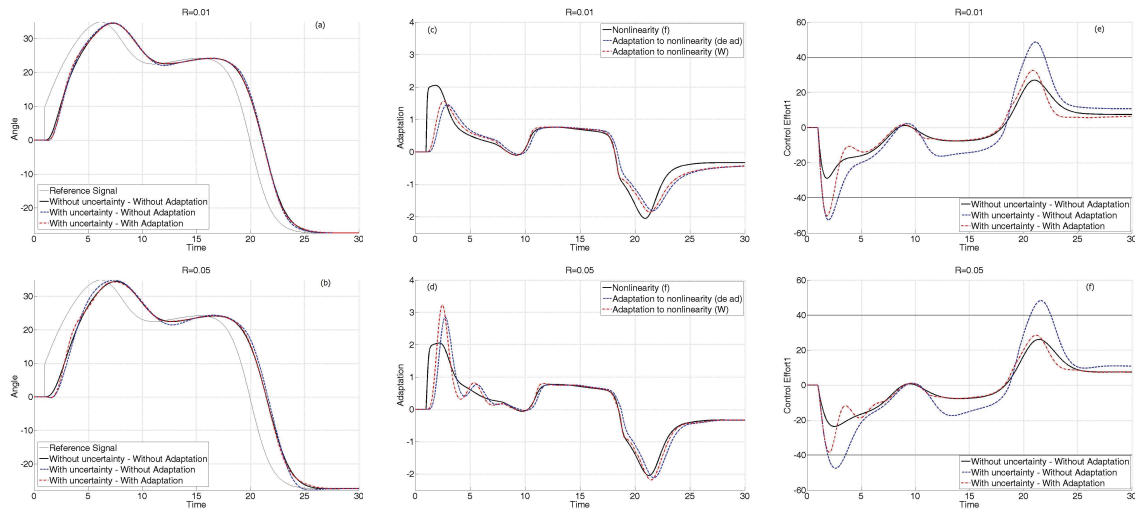


Fig. 1. System performance for $R = 0.01$ and $R = 0.05$ with or without nonlinear uncertainty and with or without adaptation to track reference signal, to adapt to nonlinearity and to use small control effort in presence of nonlinearity. (a), (b) Reference signal $r(t)$ and the angle of attack $\alpha(t)$. (c), (d) Nonlinearity $f(\alpha(t), \delta_e(t))$ and its approximation by $\delta_{e_{ad}}(t)$ and $\hat{W}(t)$. (e), (f) Control effort $C_e(t)$.

$R = 0.05$ it is overestimated. The plots in Fig. 1(e) and (f) show the PI control effort $C_e(t)$ as a function of time. The introduction of uncertainty increases the PI control effort significantly (see value of $E_{C_e}(1, 1, 0, 0)$ and $E_{\max C_e}(1, 1, 0, 0)$ in Table I). The adaptive law permits the reduction of the PI control effort needed for tracking and making it similar to the control effort signal without uncertainties. As the capability of the typical elevator deflection $\delta_e(t)$ is limited, the PI control effort should not be above a certain value. Assuming the limit of the control effort is $40 \text{ deg} \cdot \text{s}^{-1}$, we can see that the PI controller with $R = 0.01$ goes above this limit when uncertainties are present even with the adaptive law. When $R = 0.05$, the PI control effort with uncertainty is above 40 deg but gets within this limit when the adaptive law is used. The PI controller with adaptive law is efficient at tracking the reference signal and keeping the control effort low. Hence, the parameter R can be tuned depending on design constraint.

IV. SYSTEM PERFORMANCE WITH GAINS OR DELAYS

We are interested in quantifying how robust the controller is to changes in gains k_1 and k_2 and delays τ_1 and τ_2 . For this highly nonlinear system, the following study cannot be performed analytically.

The system is considered to have “stable tracking” if

$$E_{Ir}(k_1, k_2, \tau_1, \tau_2) \leq 2E_{Ir}(1, 1, 0, 0). \quad (14)$$

The system is considered to be “controllable” if

$$E_{\max C_e}(k_1, k_2, \tau_1, \tau_2) \leq 40. \quad (15)$$

The system is considered to be “controllable” and to have stable tracking if both stability indicators are satisfied. We use these three indicators to characterize whether the controller is acceptable.

A. Controller Performance for Parameters $(k_1, k_2, \tau_1, \tau_2)$

GoSUM software is used to learn the controller success indicators as a function of the 4 parameters $(k_1, k_2, \tau_1, \tau_2)$. First, the software generates 5000 samples in a 4-D compact region for $(k_1, k_2, \tau_1, \tau_2)$ using a Latinized CVT method [17] modified to be applicable for a large number of parameters. Then, 5000 runs of the control system are done at those points with outputs E_{Ir} and $E_{\max C_e}$. The resulting

TABLE I

(a) NUMBER OF STABLE POINTS AMONG THE 5000 SAMPLES.
(b) SYSTEM PERFORMANCE FOR $R = 0.01$ AND $R = 0.05$ IN THE CASE WITH AND WITHOUT NONLINEAR UNCERTAINTY AND WITH AND WITHOUT ADAPTATION

No. of stable points	$R=$	Tracking	$E_{\max C_e} < 40$	Both
No adaptation	0.01	2677	529	451
No adaptation	0.05	3846	1398	1261
Adaptation	0.01	1488	502	478
Adaptation	0.05	2165	2088	1731

(a)

Error	R	No uncert. no adapt.	uncert. no adapt.	Uncert. adapt.
E_{Ir}	0.01	0.55596	0.58271	0.55903
	0.05	0.71107	0.77588	0.71245
E_{ad}	0.01	0	4.7808	2.3058
	0.05	0	4.9115	2.0101
E_{C_e}	0.01	69.5404	118.2412	80.0759
	0.05	68.0395	118.801	72.2523
$E_{\max C_e}$	0.01	28.8522	52.5501	50.2281
	0.05	26.094	48.2841	38.4032

(b)

(5000×2) output vector is transformed to a (5000×3) binary output vector with true = -1; false = 1. The first column corresponds to the indicator in (14), the second column to the one in (15), and the third column to the third indicators. We perform runs for four different setups of the controller ($R = 0.01$ and $R = 0.05$ for the LQR controller with and without adaptive law). The interval of study for the gains includes the typical gains $(k_1, k_2) = (1, 1)$ and has a minimum value of half the typical gain and a maximum value of double the typical gain. Note that without adaptation and without uncertainties, we have an LQR controller and hence the system has gain margins $k_1 \in (0.5, +\infty]$.

The results presented here use sampling in the 4-D region $(k_1, k_2, \tau_1, \tau_2) \in [0.5; 2] \times [0.5; 2] \times [0; 0.4] \times [0; 1]$. In Table I(a), we show how many of the 5000 samples result in the indicators

TABLE II

VOLUME OF THE “4-D CUBE” MAXIMIZING Vol_{opt} AND NUMBER OF POINTS N_{true} THAT SATISFY THE CONSTRAINT. VALUES OF GAIN AND TIME DELAY MARGINS ARE GIVEN. THE DOMAIN OF STUDY IS $(\tau_1, \tau_2, k_1, k_2) \in [0; 0.4] \times [0; 1.0] \times [0.5; 2.0] \times [0.5; 2.0]$

	R	Tracking		$E_{\max C_e} < 40$		Both		
		Vol	N_{true}	Vol	N_{true}	Vol	N_{true}	
No adaptation	0.05	0.361	611	0.133	401	0.139	396	
Adaptation	0.05	0.133	618	0.164	611	0.10871	610	
No adaptation	0.01	0.158	610	0.046	374	0.050	375	
Adaptation	0.01	0.080	615	0.049	389	0.040	398	
		$\tau_{1\max}$	$\tau_{2\max}$	$\tau_{1\max}$	$\tau_{2\max}$	$\tau_{1\max}$	$\tau_{2\max}$	
No adaptation	0.05	0.399	0.802	0.203	0.505	0.211	0.657	
Adaptation	0.05	0.199	0.964	0.202	0.999	0.201	0.967	
No Adaptation	0.01	0.245	0.542	0.099	0.424	0.100	0.500	
Adaptation	0.01	0.185	0.818	0.104	0.579	0.100	0.652	
		min	max	min	max	min	max	
No adaptation	0.05	k_1	0.525	2	0.999	1.999	0.999	1.999
		k_2	0.840	1.605	0.700	2	0.796	1.798
Adaptation	0.05	k_1	0.800	2	0.836	2	0.865	1.999
		k_2	0.721	1.300	0.5	1.199	0.714	1.206
No adaptation	0.01	k_1	0.624	1.996	0.999	1.999	0.999	2
		k_2	0.756	1.622	0.733	1.832	0.800	1.801
Adaptation	0.01	k_1	0.775	1.990	0.999	1.999	0.999	1.999
		k_2	0.752	1.188	0.753	1.563	0.888	1.507

to be true (binary output is -1). First, let us look at the tracking performance alone. We also see that in the case $(k_1, k_2, \tau_1, \tau_2) = (1, 1, 0, 0)$, the tracking error is smaller for $R = 0.01$ than for $R = 0.05$. But the system with $R = 0.05$ is more robust to changes in the two gains and the two time delays than the system with $R = 0.01$. Moreover, the system with adaptation loses its tracking ability more often than the system without adaptation. Finally, in the case $(k_1, k_2, \tau_1, \tau_2) = (1, 1, 0, 0)$, the controller effort is smaller for $R = 0.05$ ($E_{C_e} = 72.25$; $E_{\max C_e} = 38.40$) than for $R = 0.01$ ($E_{C_e} = 80.07$; $E_{\max C_e} = 50.22$); moreover, only the controller with $R = 0.05$ and adaptive law satisfies $E_{\max C_e} < 40$. Tuning of the two gains and delays permits having more systems respecting these criteria. But the controller with $R = 0.05$ and adaptive law seems to be the most robust to changes in the two gains and time delays.

We also would like to know the boundary of the domain where the controller is successful. The model is learned using the SVM method in GoSUM software. Then the three binary outputs that indicate the robustness of the system can be quickly predicted using the learned model at any point $(k_1, k_2, \tau_1, \tau_2)$ in the 4-D space $[0.5; 2] \times [0.5; 2] \times [0; 0.4] \times [0; 1]$. Fig. 2 shows the 3-D boundary between the region where the parameters satisfy our criteria for both tracking and control bound and the region where they do not and was created using a uniform sampling of 101 points in each direction of the cubes $(k_1, k_2, \tau, \tau) \in [0, 2] \times [0, 2] \times [0, 0.4]$ and $(k, k, \tau_1, \tau_2) \in [0, 2] \times [0, 0.4] \times [0, 1.0]$ for the case with adaptive law and $R = 0.05$.

B. Parameters $(k_1, k_2, \tau_1, \tau_2)$ as Uncertain Parameters

To find the gain and time delay margins in the cases where k_1, k_2, τ_1 , and τ_2 are uncertain parameters, we need to find the min and max values for k_1, k_2, τ_1 , and τ_2 such that closed-loop

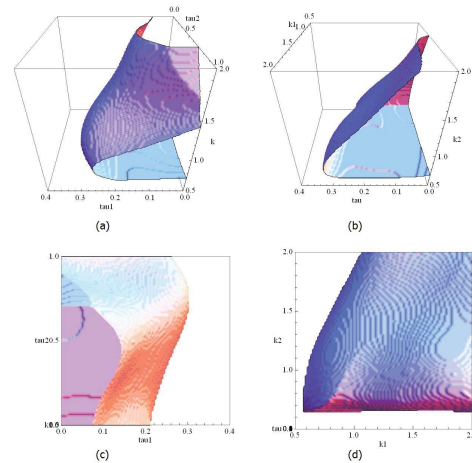


Fig. 2. Illustration of the predicted 3-D boundaries between success and failure for both tracking and control bound, depending on the gain and delays values for the case with adaptive law and $R = 0.05$. (a), (c) 3-D boundary is plotted with the constraint $k_1 = k_2 = k$. (b), (d) 3-D boundary is plotted with the constraint $\tau_1 = \tau_2 = \tau$.

stability and tracking are preserved, or the maximum volume Vol of a “4-D cube” $[k_{1\min}; k_{1\max}] \times [k_{2\min}; k_{2\max}] \times [0; \tau_{1\max}] \times [0; \tau_{2\max}]$ in $[0.5; 2] \times [0.5; 2] \times [0; 0.4] \times [0; 1]$ where each point respects the constraint of interest. A “4-D cube” is sampled using a uniform distribution of 5 points in each direction (a total of 625 points). Using the learned models, we use GoSUM to predict the binary output for the constraint of interest for each sample points. We then count the number of samples N_{true} that satisfies the constraint of interest.

The hard constraint $N_{\text{true}} = 625$ is replaced with a soft one $(N_{\text{true}}/625)^{10}$. Using GoSUM mesh adaptive direct search [18], we find the maximum value of

$$\text{Vol}_{opt} = \text{Vol} \left(\frac{N_{\text{true}}}{625} \right)^{10}. \quad (16)$$

The results are given in Table II. For the tracking stability, the highest volume is obtained with no adaptation and $R = 0.05$. For this method, $k_{1\min}$ is the lowest and $\tau_{1\max}$ is the highest. We can see that $k_{1\max}$ is not influenced by the method used. The boundary values $k_{1\min}$ and $\tau_{2\max}$ are higher for methods using the adaptive law than those not using it, while the opposite is true for $k_{2\min}$, $k_{2\max}$, and $\tau_{1\max}$. The controller with adaptive law and $R = 0.05$ is the only one satisfying the constraint that $E_{\max} C_e < 40$ for $(k_1, k_2, \tau_1, \tau_2) = (1, 1, 0, 0)$ (see Fig. 1). So, only for this case is the $N_{\text{true}} \approx 625$.

We tested the results given for $R = 0.05$ for the method without adaptation by running the full controller system simulation on the 625 points sampling the “4-D cube.” The accuracy of the learned model is defined as the number of correct points divided by the total number of points in the “4-D cube.” All the points inside the cube satisfy the tracking performance indicator. On the sides of the “4-D cube,” 20 sample points did not satisfy the tracking performance indicator. In this first test, the learned model has 99.04% accuracy. We also tested the results given for $R = 0.05$ for the method with adaptation. All the points inside the cube satisfy the tracking performance indicator. On the side of the “4-D cube,” 17 sample points did not satisfy both indicators. In this second test, the learned model has 99.68% accuracy. The learned model accuracy at the boundary between stable and unstable region can be improved by using more sample points.

The use of a learned model permits a significant decrease of computational time to obtain the gain and delay margins. On average, a simulation of the system takes 2 s of CPU time. We sampled the domain with 5000 points which took about 2 h and 30 min of CPU time; then using the learned model, it only took 0.4 s to predict the values on the 625 points of the “4-D cube.” Assuming the optimization needs 200 iterations, 200×625 predictions are needed, which takes about 70 h of CPU time if solving the controller system. By using the learned model it only takes 1 min 20 s of CPU time. Hence the computational time was decreased by more than a factor of 20.

V. CONCLUSION

Both adaptive and nonadaptive controllers for an aircraft pitch dynamics were considered. The system had uncertain nonlinearity in control and unknown gain and time delays. A performance and robustness study of the controller was performed numerically. The controller performance with and without adaptation was analyzed in a 4-D region of the unknown gain and time delays space, and the gain and time delay stability margins were calculated. Using the sampling, learning, prediction, and optimization capabilities of GoSUM [14] the performance study described here was possible with only 5000 runs of the model and hence with low computational time compared to current validation methods such as Monte Carlo sampling.

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