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Uncertainty in the Energy Dynamics of Commercial Office Buildings

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Abstract— **Whole-building energy models take information about the structure of a building, its equipment (electrical loads, lights, conditioning equipment, etc.), and disturbances (people, weather) and predict its year long comfort and energy performance. Both commercial and freely available tools are available for performing these time-domain simulations, which are used for design trade studies and more frequently to check for energy consumption and comfort compliance. These models require hundreds of assumptions as input when it comes to parameterizing the building model. Previous studies have investigated how predictions are influenced by these assumptions and which of the parameters are critical to year-long calculations. In this paper we extend this approach to investigate how parametric uncertainty influences uncertainty in the energy dynamics within a building. We provide a case study that investigates an office building by extracting dynamic information out of an EnergyPlus model, and supplies this information to an automatically generated analytical thermal network model. We conclude with a control-oriented frequency-based robustness assessment as well as a study of how uncertainty influences the network structure of the building by investigating the spectral gap of its graph Laplacian.**

I. INTRODUCTION

The purpose of the built environment is to provide comfortable and productive shelter from the elements, while doing this contributes to a large portion of global energy expenditures and subsequent air pollution. In the past few decades, there has been significant advances in construction materials and practices to maximize both energy efficiency and occupant comfort. However, regardless of how well a building is built, its operation and control plays an ultimate role in its performance. In order to achieve optimal operation of a building, model-based methods are needed as an integrated design tool for controller synthesis, realization, and real-time implementation (e.g. model predictive control).

There are many different types of models for buildings and their equipment [1], in terms of control analysis, models can be classified into three different categories; local equipment models, whole building energy models, thermal network models. In many cases, control analysis is performed locally at the equipment level (e.g. [2], or [3]), with the remainder of the building acting as a disturbance. This works well for smaller buildings with a limited number of equipment, as the size of the building grows and the number of equipment multiplies, interactive effects make this less effective.

In the case of larger buildings, whole-building energy models (e.g. EnergyPlus [4] or TRNSYS [5]) are used to predict energy and comfort at long time scales (e.g. monthly, seasonally, yearly). Although their intention is primarily for energy prediction, in some studies, whole-building energy models are used for the detailed design of a control system (e.g. proportional and integral coefficients [6]), while it is unclear if the dynamics in the closed-loop model are accurate enough to make these sort of conclusions. On the other hand, the use of these models for setpoint optimization has been successful as these models are more accurate at these time scales [7].

Because of the lack of dynamic fidelity of the controllers in most whole-building energy models, control-oriented analysis of building energy systems is usually performed with smaller hand-generated thermal network models. A thermal network model describes the physics of a building as a network of resistors and capacitors. Because these models are hand generated, case studies are often performed on buildings that are small in size and contain only limited numbers of equipment. There has been successful use of these models, particularly in the model predictive control community (e.g. [8] and [9])

In this paper we use a thermal network model that extracts information from a whole-building energy model. The thermal network model is derived in parallel with the construction of a whole-building EnergyPlus model. Parameters from the EnergyPlus model are then extracted after its compilation and put into the smaller model (no system identification is performed).

The models discussed above are derived from many assumptions about how a building is constructed, operated, and used by its occupants. Initial values for these assumptions often come from design documents or observed behavior, while in fact, a building is not always built or used exactly how it is designed. Uncertainty propagation is used to quantify how this uncertainty influences model predictions and sensitivity analysis is used to identify critical parameters to this variance ([10], [11], and [12]). Even though uncertainty quantification for whole-building building energy models has reached a relative maturity, typical metrics that are investigated are year-long or seasonal energy predictions and occasionally persistent peak demand (e.g. over an hour, not necessarily a control transient). There hasn't been an attempt to quantify uncertainty in dynamics or other control-oriented implications to parametric uncertainty in whole building energy models.

In this paper, we tie together a modeling approach that extracts dynamics from large whole-building energy models, and using uncertainty propagation methods, quantify how parametric uncertainty influences control-oriented aspects of the building energy dynamics.

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II. MODEL DEVELOPMENT

In order to generate the reduced order model, parameters and operating point information from an EnergyPlus model were extracted and inserted into a set of analytical equations that describe the building dynamics. These dynamics include heat balances in each zone and in all constructions. For each construction surface, the balance is performed considering long-wave radiation between other surfaces, convection to the zone, and conduction through the construction. The following are assumptions and special considerations for this model:

- A1 Air and surface temperatures are both spatially uniform. A construction can contain multiple surfaces (e.g. a wall with a window).
- A2 External constructions exchange heat by convection with outdoor air. Influence from wind or solar radiation is only considered through the operating point.
- A3 One-dimensional conduction occurs between two surfaces of a construction which is homogenous within each layer, but can be composed of many layers.
- A4 Internal loads which are not state-dependent are considered as disturbances, this includes both latent heat and influences from short wave radiation.
- A5 Only sensible heat balances are considered, the air has a constant density ($\rho = 1.2 \left[kg/m^3\right]$) and specific heat ($c_p = 1000 \left[J/kgK\right]$).
- A6 The case study in this paper does not have internal partitions that allow intra-zone mixing, and therefore convective transfer between zones is omitted for this model (it can be added relatively easily). Infiltration from the external environment is also considered negligible.

Again, it should be noted that most of the physics that are explicitly omitted in the formulation of the linear reduced order model are captured in operating point conditions. The equations relevant to each heat balance in the model are presented below, while further information about this model is available in [13] or in the freely available documentation to EnergyPlus. Although there are different ways to model some aspects of building energy dynamics, we restrict the modeling methodology to parallel the EnergyPlus approach because we are taking parameters and results from this software for the operating point of our model.

A. Zone Air Balance

The thermal balance is performed on a zone by zone basis (a zone is either a single room or a group of rooms that behave similarly and are lumped into a single volume). In each of these zones, the thermodynamics are uniform across the volume and are modeled as

$$C_z \frac{dT_z}{dt} = \sum_{i=1}^{N_{\text{surfaces}}} \dot{Q}_{\text{conv}_i} + \dot{Q}_{\text{HVAC}_z}, \tag{1}$$

where $C_z \frac{dT_z}{dt}$ is energy stored in air in zone z, $\dot{Q}_{\text{conv}_i} = h_i A_i (T_{si} - T_z)$ is the heat exchanged between air and each of the zone surfaces, and $\dot{Q}_{\text{HVAC}_z} = \dot{m}_z c_p (T_{sup_z} - T_z)$ is heat added from the HVAC system. In this equation, C_z is thermal capacitance; $C_z = \alpha \rho V c_p$, where α is a sensible heat capacity multiplier and ρ and c_p are defined above. The remaining parameters are due to specifics of the layout of the building (surface area A_i, and volume V_i) which are typically generated using architectural-like software and stored in compiled files from the EnergyPlus model. The variable h_i (the surface to air heat transfer coefficient), and $\dot{m}_z$ (mass flow rate of heating and cooling) are both taken as an operating point variables as described in Section IV. In Equation 1, the heating and cooling influence through $\dot{Q}_{\text{HVAC}_z}$ is modeled as a Constant Air Volume (CAV) system where the supply air temperature T_{sup} is a control variable and hence an input to this equation.

B. Surface Balance

The second part of the model, which interacts directly with the zone temperatures, is the surface balance that captures both the capacitive and resistive influences of the structure of the building. There are three types of constructions in this model; capacitive walls (both interior and exterior), capacitive internal constructions (that capture effects like furniture), and resistive constructions (the windows have no dynamics and are purely resistive). The surface balance manages all thermal influences that occur on the surfaces of these constructions within the building. This balance equates conduction through the building constructions, convection to the zone air ($\dot{Q}_{\text{conv}_i}$), and longwave radiant interchange. The resulting surface balance for the ith surface is

$$\dot{Q}_{\text{LWS}_i} + \dot{Q}_{\text{cond}_i} + \dot{Q}_{\text{conv}_i} = 0 \tag{2}$$

where $\dot{Q}_{\text{LWS}_i}$ is long wave radiation between surfaces, $\dot{Q}_{\text{cond}_i}$ is conduction through a construction, and $\dot{Q}_{\text{conv}}$ is heat exchanged between air and wall (the same term as in Equation 1).

Heat conduction through a construction

The heat conduction through construction is either purely resistive or purely capacitive, and in some cases a mixture of both (layered constructions). The approach for handling one dimensional heat conduction through the capacitive constructions is to discretize them using a central difference scheme (this was adopted from the EnergyPlus approach). The number of nodes is determined by each layers Fourier number and is restricted to be no less than six. In the EnergyPlus software, a state space system is then discretized in time (e.g. using the z-transform), and numerous heuristic modifiers are placed on these dynamics to ensure a robust solution. We found this to be unnecessary and simply created a state space model for each construction using a series representation of heat conduction through multiple layers. An example set of differential equations for conduction through a single layer in a wall between surfaces si and sj that has been divided into M nodes are

$$C_1\dot{T}_1 = h_iA_i(T_{si} - T_1) + \frac{T_2 - T_1}{R_1} \quad (3)$$

$$C_2\dot{T}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_3 - T_2}{R_2} \quad (4)$$

$$\vdots$$

$$C_M\dot{T}_M = h_jA_j(T_{sj} - T_M) + \frac{T_{M-1} - T_M}{R_M}, \quad (5)$$

where R_i is the thermal resistance divided by the incremental thickness of the layer of the wall (Δx), and C_i is the heat capacity of the layer of the wall multiplied by Δx.

The purely resistive constructions in this model capture the influence of the fenestration (windows) and their inside and outside temperatures. Because of the lack of dynamics that relate these two temperatures, proper inclusion of both of these algebraic variables would confine the dynamics to a manifold resulting in a differential algebraic system of equations. For simplicity in analysis, we choose to omit the outdoor air temperature (on windows only) and model the inside surface temperature of the windows as a exogenous influence to the model.

Long wave radiation between surfaces

The long wave radiation between each surface is calculated as

$$\dot{Q}_{\mathrm{LWS}_i} = \sum_{j=1}^{N_{\mathrm{surfaces}}} \sigma F_{i,j}(T_{sj}^4 - T_{si}^4), \quad (6)$$

where $F_{i,j}$ is the Script-F exchange coefficient, and $\sigma = 5.6697 \times 10^{-8}\left[\frac{W}{m^2K^4}\right]$. The Script-F factor includes view factor and area information for all surfaces that can *see* the i^{th} surface, including factors for reflection, absorptions and re-emissions through the coefficients [14]. The emissivity, ratio of surface areas, and absorption is all part of this factor. These factors for all surfaces in the building are calculated at compilation time for the EnergyPlus model and are available to the user.

Even with complete information about how long wave radiant transfer occurs between surfaces in the building, it is still necessary to linearize Equation 6 for control analysis. There are multiple ways to perform this linearization [15], for this model the linearizing radiation transfer coefficient is used

$$h_{\mathrm{rad}_{i,j}} = 4\left(\frac{\overline{T}_{sj} - \overline{T}_{si}}{2}\right)^3, \quad (7)$$

resulting in

$$\dot{Q}_{\mathrm{LWS}_i} = \sum_{j=1}^{N_{\mathrm{surfaces}}} h_{\mathrm{rad}_{i,j}}\sigma F_{i,j}(T_{sj} - T_{si}). \quad (8)$$

This approach is very similar to the *T-star* network model used in another common building energy modeling program TRNSYS [16]. All terms in Equation 8 are either state variables (the surface temperatures), variables solved at the time the model is compiled ($F_{i,j}$) or operating point variables $\overline{T}$.

As introduced above, the fundamental equations within the building are not very complex. The complexity arises due to the vast amount of information relating each of the static and dynamic elements and their interconnected repetition throughout different zones. It is here that we utilize the organized structure of the compiled EnergyPlus files to systematically access this information and subsequently introduce it into the reduced model. These files include information related to interaction between surfaces, and zones, as well as lumped information like resistive and capacitive quantities for the constructions. A wrapper function was created to extract this information and assemble a linear time invariant state space system of the form

$$\dot{x} = A(x_0, p)x + B_u(x_0)u + B_w(x_0, p)w \quad (9)$$

$$y = Cx \quad (10)$$

where x, u, and w are time dependent vectors. In particular, x are the state variables, x_0 are operating point variables, and p are static parameters. The input matrices pertain to the HVAC flows (B_u) and disturbance from outdoor air (w_1), ground temperature (w_2), and inside surface temperature of the windows (w_3).

III. CASE STUDY

To test this approach for control oriented analysis of whole building energy dynamics, a United States Department of Energy (DOE) EnergyPlus Benchmark Model was used [17]. The DOE benchmark model suite contains 16 models that represent approximately 70% of commercial building stock in the United States. The models are then organized so that each one can be simulated at different locations in the United States (using typical meteorological year weather data for each of these locations).

The model studied in this paper is a new construction medium office building located in Las Vegas, Nevada. This building has three floors with 4,982 m^2 (53,628 ft^2) of floor area. The building is a rectangular cube (aspect ratio 1.5), with 33% window to wall ratio, and is zoned with 5 zones per floor (one central zone and one zone for each perimeter side of the building). An unconditioned plenum is also modeled on top of each floor. Further information about the building, its construction and equipment can be found in [13] or on the Department of Energy website [18].

EnergyPlus version 7.0.0.036 was used to gain parametric and operating point information using the Structured Query Language (SQL) compiled output of the model. The reduced order model was coded in a way that different thermodynamic phenomena can be enabled or disabled to study both input/output influences as well as parametric influence.

In total, the model contains 18 zones (we choose zone air temperature as an output, making 18 outputs), and 10 different types of constructions that are utilized in the different zones. Once assembled, there are 155 surfaces for heat exchange, and upon placing all of these interacting subsystems together, the model contains 1056 state variables (note that the number of state variables changes slightly during the uncertainty analysis as the properties of construction layers change).

IV. Design Day Operating Point

Nominal values are needed for the convective heat transfer coefficients (h_i in Equation 2) as well as the surface temperatures for the linearized radiation ($\overline{T}$ in Equation 8), and the mass flow rates that supply the HVAC air to each zone $\dot{m}_z$. To gain this information, two design days were simulated using EnergyPlus. A design day is used in the industry to characterize worst case demands on the equipment of the building which is then used to size this equipment.

The Las Vegas Annual Heating 99.6% condition set on January 21st (denoted DD1 in data presented later), and the Annual Cooling 0.4% conditions set on July 21st (DD2) were used for design day conditions. The percentage indicates the percentage in hours that the ambient temperature is expected to exceed these conditions in a year. The operating point is obtained by averaging data that is calculated at 15 minute intervals throughout the design day. Further information about the design day conditions can be found in [19], and the data from the design day calculation for the model in this paper can be found in [13].

V. Uncertainty Analysis

To assess how uncertainty influences the building dynamics, a deterministic sampling approach was used to perturb nearly all parameters in the energy model. The deterministic approach was used because its convergence bound $\mathcal{O}(N^{-1}(\log N)^{p-1})$ (N is the number of iterations, p is the dimension of parameter space) is faster than the standard random Monte-Carlo approach of $\mathcal{O}(N^{-\frac{1}{2}})$, and coverage in the volume is uniformly ergodic [20]. The software used for calculating these samples is available at [21].

To perform the uncertainty analysis, the nominal input file was parsed to identify all numerical parameters that pertain to the construction, usage, and operation of the building. Architectural parameters (size, layout, orientation), and polynomial coefficients on some of the equipment curves were omitted. Once this parsing was performed, 771 parameters in the model were identified. The top three classes of these parameters are; schedule values (172), plug loads and elevator consumption properties (129), and construction material properties (99).

A range and distribution type was chosen for each of the parameters and each were simultaneously sampled 5000 times. Parameters with nonzero nominal value were sampled using a uniform distribution with a range of $\pm 25\%$ of the nominal value. Parameters with a zero nominal value were sampled using an exponential distribution so that its mean is closer to the nominal value. Care was taken to consider bounds on variables (e.g. fractional parameters) as well as constrained parameters (e.g. $p_1 + p_2 < 1$). Simulations were parallelized and simulated on a multi-core desktop computer, at about 3 minutes per simulation, the computation time did not require multi-processor computational power.

A. Uncertainty in Controlled Performance

In order to gain an understanding of how parameter uncertainty influences controlled performance of the building, the sensitivity function between disturbances and controlled output of the plant was calculated for each parameter set. The sensitivity function is defined as

$$S(s) = \frac{G_w(s)}{1 + G_u(s)K(s)}, \tag{11}$$

where the transfer function between control input and output is $G_u(s) = C(sI - A)B_u$ and between the disturbances and the output is $G_w(s) = C(sI - A)B_w$. To close the loop, a decoupled proportional-integral (PI) controller for each conditioned zone was generated using

$$K_z(s) = \frac{k_p s + k_i}{s}, \tag{12}$$

where $k_p = 10$ and $k_i = 1/500$ are coefficients for the controller. Note that $K(s)$ in Equation 11 is a matrix of 15 uncoupled PI controllers with equivalent coefficients (this is not an uncommon strategy in building system control).

The magnitude of the frequency response between disturbances (w in Equation 9) and the 18 zone outputs is presented in Figures 1, 2, and 3. In each figure, the frequency response for all uncertain model realizations was calculated and then bands of single standard deviation from the mean are filled with color. In each plot, zones that behave similarly are grouped by different colors. In addition to this, two different operating points are presented, one for each design day calculation.

In Figure 1, the closed loop sensitivity between outdoor air and zone temperatures is presented. In this figure it is clear that there is little sensitivity to outdoor air in the conditioned zones. The plena are exposed to outdoor air disturbances but have no HVAC conditioning so they are more responsive. In addition to this, the core zones behave differently than the external zones because they are not exposed to outdoor air through exterior walls.

In Figure 2 the response between ground temperatures and zone air temperatures illustrates that again, the plenum temperatures are more sensitive to external disturbance (there is also some variability due to how far the plenum is from the ground as well). The slightly larger variation in low frequency dynamics for the first design day (DD1) is likely due to surface heat transfer coefficients being larger for this day.

The response between inside window surface temperatures and controlled zone air temperatures is presented in Figure 3. In this figure, it is evident that the zones with fenestration respond to window surface temperature more than the core zones. Although the plenum zones contain no windows, they respond from conduction through neighboring zones (the plena have no feedback control to reduce this sensitivity).

The controlled response between the setpoint and zone air temperatures is presented in Figure 4. The variation between zones is due to the size of the zone (the amount of air volume

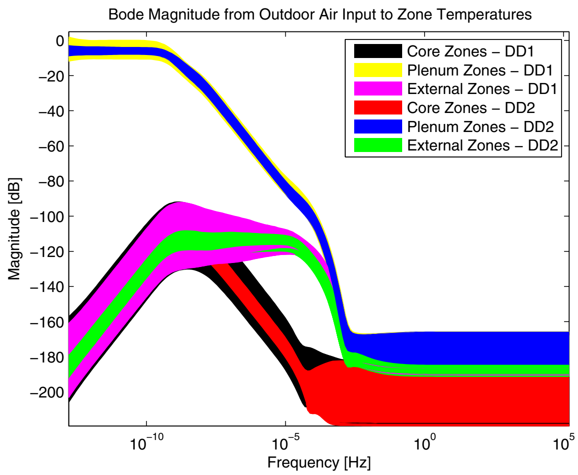

Fig. 1. Sensitivity of closed loop performance between outside air temperature and zone air temperature considering parametric uncertainty.

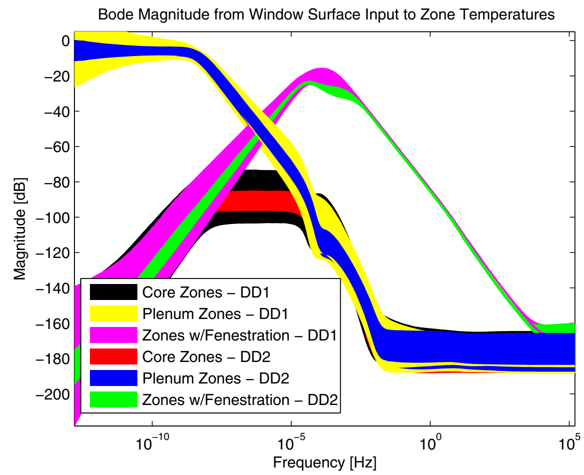

Fig. 3. Sensitivity of closed loop performance between inside window temperature and zone air temperature considering parametric uncertainty.

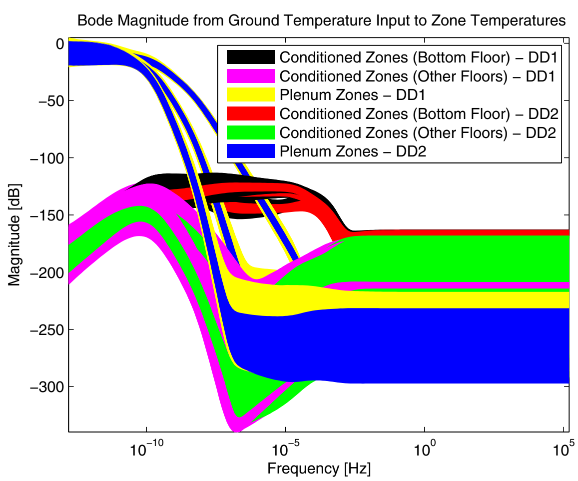

Fig. 2. Sensitivity of closed loop performance between ground temperature and zone air temperature considering parametric uncertainty.

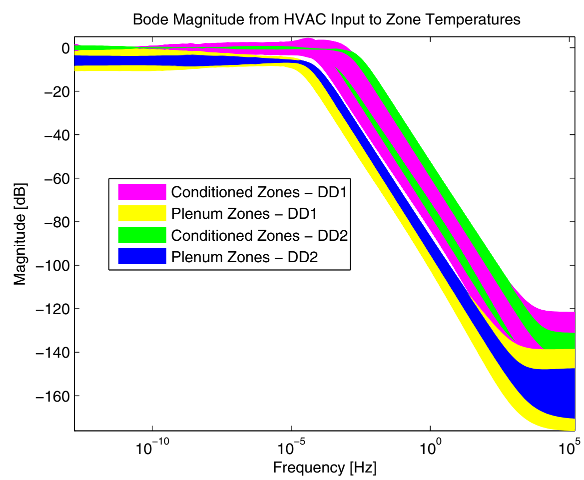

Fig. 4. Response function between setpoint temperature and zone air temperatures considering parametric uncertainty.

changes the time constant) as well as the nominal HVAC flow calculated at each design day. In addition to this, uncertainty in these sensitivity functions confirms what is seen in practice; a common controller may lose its performance based on uncertainty in the design, construction, operation, and usage of a building.

B. Uncertainty in the Interaction Dynamics

Since buildings are large systems of interconnected subsystems, it is interesting to better understand how different elements of the building influence each other. Capturing these interactions leads to a better understanding of how coupling between these subsystems influences the performance of the building as a whole, which is useful for building-wide controller synthesis and model reduction. To gain a better understanding of this coupling, we define the normalized adjacency matrix as

$$W_{\text{Norm}}(i,j) = \frac{\|A(i,j)\|}{\sum_{k=1}^{N} \|A(i,k)\|}, \tag{13}$$

where A is size $N \times N$ state space matrix from Equation 9. This matrix is then symmetrized [22]

$$W_{\text{sym}} = \frac{1}{2}(W_{\text{Norm}} + W_{\text{Norm}}^T), \tag{14}$$

and the Laplacian is defined as

$$L = deg(W_{\text{sym}}) - W_{\text{sym}}, \tag{15}$$

where $deg()$ is the degree matrix. In this formulation, the first eigenvalue of L is always zero and the second eigenvalue λ_2 is the spectral gap. The spectral gap offers a notion of mixing, quantifying how interconnectedness and the presence of coherent clusters within the dynamics. Small values of λ_2 indicate *bottlenecks* or neighborhoods that can be clustered.

Figure 5 presents uncertainty in the spectral gap where it is evident that the spectral gap does not significantly change due to parametric uncertainty or the operating point of the model. This is an important finding as it shows that the topology of the system is robust to meaningful parametric perturbation.

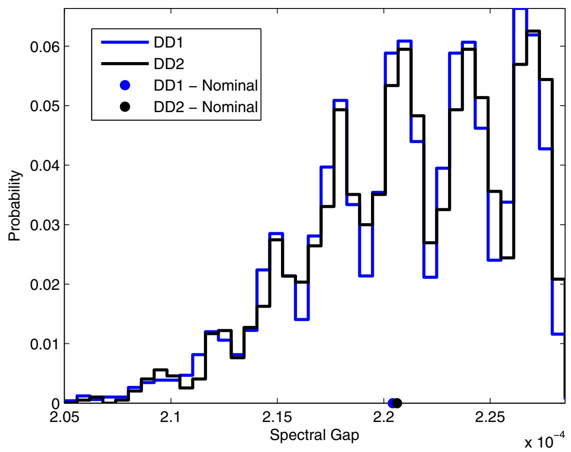

Fig. 5. Uncertainty of the spectral gap of the graph Laplacian for two different design day conditions and uncertainty in 771 physical model parameters.

Sensitivity analysis was performed on the spectral gap data to identify critical parameters that were driving its uncertainty. It was found that parameters associated with internal mass (e.g. furniture) influence the variation in spectral gap the most. The size of Laplacian is highly correlated to these parameters as the number of dynamic states varies based on parameters of these constructions (e.g. thickness). This explains why the distribution is *toothed* - this response is due to different state space matrix sizes.

VI. SUMMARY

In this paper we presented an approach to leverage the detailed information in a compiled whole-building energy model to construct a reduced order control-oriented thermal network model. Using this approach, we quantified how realistic uncertainties in the design, construction, operation, and use of building influences its dynamic response. In investigating the uncertain sensitivity function, the loss in performance when a single controller is used for a realistic perturbation in the building dynamics has been quantified. In addition to this, uncertainty in the network topology has been investigated, where it has been shown that this topology is robust to reasonable uncertainty in the parameters of the model. This is important as it illustrates that reduced order analysis and model reduction should be robust to these uncertainties.

VII. ACKNOWLEDGMENTS

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