CREATING ZONING APPROXIMATIONS TO BUILDING ENERGY MODELS USING THE KOOPMAN OPERATOR

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ABSTRACT

As the scope of building construction increases and designs become more integrated, building energy models have found widespread use in evaluating building performance. Despite the growing sophistication of building modelling tools, errors can arise from the approximations that are made during model creation. This paper addresses model zoning, i.e., how the volume of a building is divided into regions where properties are assumed to be uniform. Zoning is important during the creation of a model because the accuracy of prediction from simulating a model reduces when dissimilar zones are lumped together. In this paper, a systematic approach to creating zoning approximations is introduced to investigate the effect of zoning on simulation accuracy. Applying the Koopman operator, an infinite-dimensional, linear operator that captures nonlinear, finite-dimensional dynamics without linearization, a detailed building model is studied. Using the Koopman operator, the temperature history of rooms produced by a building simulation can be decomposed into Koopman modes. These modes identify dynamically significant behavior which will form a basis for the creation of zoning approximations. An implementation of this technique is illustrated in a building model of an actual building designed with both mechanical and natural conditioning.

INTRODUCTION

Development of building energy models has been ongoing for several decades, and model-based analysis has found use in multiple areas of the building systems field. Applications of model-based analysis include proof-of-concept feasibility studies that may be performed during the design phase of building construction (Binks 2011; Gross and Hu 2011; Hes et al. 2011), benchmarking energy efficiency such as performed for obtaining LEED certification (UCSGBC 2011), and control or retrofit studies after building construction has been completed (Chahwane et al. 2011; Kabele et al. 2011). As a result of the variety of uses that exist for building models, the field is becoming increasingly interdisciplinary with interests from architects to engineers from industry and academia with varying technical backgrounds.

The relevant dynamics of a building can be described by a system of partial differential equations (PDEs). This system is often too time consuming to fully simulate for long durations of time (e.g., a yearlong period), therefore approximations are normally made. Examples include one-dimensional heat transfer through walls and floors and uniform material properties. Thus the original PDEs are transformed into a lumped parameter system which takes less time to solve numerically. Building simulation environments such as EnergyPlus, TRNSYS, ESP-r, use these assumptions when predicting building performance and numerous studies exist that reinforce the validity of these simulation environments and their assumptions for predicting building heat transfer(Bradley, Kummert, and McDowell 2004; US:DOE 2012). Generally, the most detailed models these programs are capable of producing are those in which each room of a building is treated as an individual zone with uniform properties. However in practice, a model where each room is individually zoned can be time consuming to develop. To reduce the time necessary to develop a model, the creator of the model devises zoning approximations where the properties of multiple rooms are lumped together into a single zone. From this simplification, the model representation of a building can contain fewer zones than there are physical rooms. Since the number of parameters becomes increasingly difficult to manage as model complexity grows, and complex models require more time to fully simulate, zoning approximations are made to mitigate these problems. Depending on the background and field of the modeler, different approximations may be deemed acceptable. Currently, the lumping of rooms is usually performed heuristically and based on some similarity between the lumped rooms (e.g. similar internal loads, shared HVAC components, etc.).

Several studies exist describing the effects that zoning approximations have on a building model. In (O'Brien, Athienitis, and Kesik 2011), the sensitivity to zoning of a 5 room model is explored, and the author shows that models with too few thermal zones can under-predict energy usage and comfort by over-predicting the rate of air mixing caused when rooms are lumped together. A technique for model order reduction of building models is presented in (Deng et al. 2010) which creates zoning approxima-

tions based on the time constants of rooms and their walls. These approximations preserve the physical interpretation of the building model bringing insight to the behavior of zones through control of the aggressiveness of the reduction scheme. This approach however requires knowledge of the equations describing the temperature evolution of the building model. When using building simulation environments, the thermal balance is not explicitly available to the user. The approach for creating zoning approximations presented in this paper is based off of observations of temperature with no knowledge of the equations describing the thermal balance of the building model.

In this paper, zoning approximations of building models are systematically determined (utilizing the Koopman operator). The Koopman operator is a linear, infinite-dimensional operator that captures nonlinear, finite dimensional dynamics without linearization. Using properties of this operator, dominant modes of thermal behavior can be extracted from a building simulation. The Koopman operator has previously been used to study the thermal behavior of other buildings. In (Eisenhower et al. 2010), an EnergyPlus model of a building was partially calibrated to measured data by comparing the modes produced by the model to that from measured temperature data. With information provided by these modes, one can determine when different zones of a building simulation are dynamically similar suggesting that they can be combined into a single effective zone with minimal effect on model prediction.

Besides zoning approximations, additional assumptions are made by modelers during the creation of a building model such as defining internal loads and operating schedules. This process can be error prone and inconsistent creating uncertainties which are not negligible (Tupper et al. 2011). The model studied in this paper does not incorporate any internal loads or operating schedules. This is however, not a limitation of the method, and internal loads and schedules may be included in future studies. For illustrative purposes, focus is instead given to thermal influences of the simulated weather on the building model when evaluating zoning approximations, and how accuracy decreases as a model is more coarsely zoned. Accurately capturing the environment's influence on the heat transfer of a building is important because inaccuracies from a coarsely zoned building model can compound with the imprecisions in defining internal loads, schedules, and other model parameters.

The remainder of this paper is organized as follows: in the following section, a brief overview of the Koopman operator is given. Using the Koopman operator, a detailed building model is analyzed, and zoning approximations are created based off of similarities between adjacent zones which are present from the spectrum of the opera-

tor. The paper is concluded with a summary of zoning approximations created, and their effect on model performance.

THE KOOPMAN OPERATOR

To introduce the Koopman operator, consider the evolution of a nonlinear dynamical system given by

$$\mathbf{x}(t+1) = F(\mathbf{x}(t)) \tag{1}$$

where $\mathbf{x} \in M$ are the state space variables belonging to a finite, but multi-dimensional space M, and $F : M \to M$ maps the variables at time t to time $t+1$. The Koopman operator U is a linear operator that acts on M in the following manner: for $g : M \to \mathbb{R}$, where g is a function describing observations of the state space variables, U maps g to a new function Ug given by

$$Ug(\mathbf{x}) = g(F(\mathbf{x}(t))) = g(\mathbf{x}(t+1)). \tag{2}$$

While the dynamical system may be nonlinear and evolve on a finite-dimensional space, the Koopman operator is linear, but infinite-dimensional.

The Koopman operator describes the evolution of an observable one step in time. For example, in the case of a homogeneous linear time-invariant system with the state vector as the observable, the Koopman operator can explicitly be written as the state-transformation matrix for a fixed time-step. In more complex systems, the Koopman operator is not as easily expressed.

Because the Koopman operator is linear, its eigenfunctions and eigenvalues are defined as follows: for eigenfunction $\psi_k : M \to \mathbb{C}$ and constant eigenvalues $\lambda_k \in \mathbb{C}$

$$U\psi_k(\mathbf{x}) = \lambda_k \psi_k(\mathbf{x}). \quad k = 1, 2, \dots \tag{3}$$

Vector-valued observables, $g : M \to \mathbb{R}^n$, can be expressed in terms of eigenfunctions and eigenvectors of the Koopman operator by

$$g(\mathbf{x}) = \sum_{k=1}^{\infty} \lambda_k \psi_k(\mathbf{x}) v_k. \tag{4}$$

We assume $g(\mathbf{x})$ is in the span of eigenfunctions which is true if if the initial condition $\mathbf{x}_0$ is on any attractor. In Eq.4, $\{v_k\}_{k=1}^{\infty}$ are a set of vectors called Koopman modes, and are coefficients of the projections of observables onto the eigenfunctions of the Koopman operator. Koopman modes describe the dynamics of observables at different frequencies (proportional to λ_k), and will be the basis for creating zoning approximations described later in this paper.

In (Mezic and Banaszuk 2004), a relationship between Fourier analysis and Koopman modes is identified. There are several methods available for calculating Koopman modes such as using the Arnoldi algorithm (Susuki and Mezic 2010), or by using harmonic averages of the spatial field (Mezic 2005). When observables are periodic, the decomposition can be computed using discreet Fourier transformation (Rowley et al. 2009) (as is the case for a yearlong building simulation). For more information about model decomposition using the Koopman operator, refer to the references above

In this paper, the Koopman operator is utilized in creating zoning approximations by analyzing the amplitude and phase of Koopman modes that are largest in magnitude. The observables used in this paper to calculate Koopman modes are the temperatures of building zones. These modes help illustrate characteristic patterns of temperature behavior that are intrinsic to the design and simulated weather of the building. By examining the modes that reflect the most dominant heat transfer characteristics of the building model, zoning approximations can be defined to create simplified models that aim to best reproduce the temperature behavior of the original model for the simulated conditions.

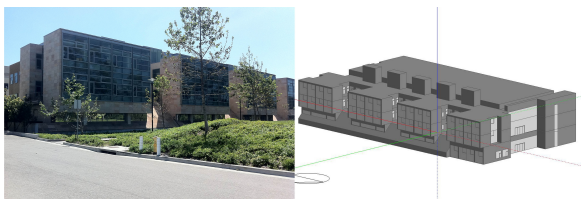## THE ENGINEERING SCIENCE BUILDING

Figure 1: Building (Left) and EnergyPlus model (Right)

The Engineering Science Building (ESB) was constructed in 2003 at the University of California, Santa Barbara. The building is 80,500 square foot and used mainly for microelectromechanical systems (MEMS) research. The ESB includes naturally ventilated office areas, research laboratories, and a class 100/1000 clean room. Because of air quality requirements on room temperature and cleanliness, the clean room and research laboratories are continuously ventilated. Due to the level of performance maintained by the heating, ventilation, and air conditioning (HVAC) system, the ESB is one of the largest users of energy on the campus consuming approximately 7,000,000 kWh of energy annually.

A model of the ESB was created in DesignBuilder using EnergyPlus version 6.0.0.023 from available design drawings. The simulated environment was generated using Santa Barbara Municipal Airport TMY3 (typical meteorological year) weather data. The ESB was chosen for

analysis because its floor plan contains both mechanically and naturally ventilated areas that serve a variety space usages (i.e., offices, laboratories, clean room). Although no HVAC system is incorporated in the model, the layout of rooms are distributed differently with influences from the HVAC system design depending on the type of space usage. Office areas contain smaller rooms that are more densely positioned while in contrast the research laboratories and clean rooms areas contain much larger rooms. This allows the effect of zoning to be studied for each of these space usages. In the model, each room is represented as its own thermal zone. Table 1 summarizes some of the defining features of the initial EnergyPlus model. This initial model contains 191 zones, with a time-step duration of 15 minutes. The model takes about 45 minutes to simulate on computer with 2.53 GHz CPU.

KOOPMAN ANALYSIS OF THE ENERGYPLUS MODEL

In this section, Koopman modes will be studied to understand the thermal behavior of the detailed EnergyPlus model. We will first discuss the spectrum of the Koopman operator and identify significant Koopman modes and the frequencies at which they occur. Characteristics of these modes will illustrate trends, at various time scales, that occur in the in the thermal behavior of the detailed model. Using similarities between these modes, zoning approximations will be created by grouping adjacent zones that express themselves similarly across the modes being considered. These approximations allow the simplified models to be created which attempt to best capture the temperature behavior of the original detailed model but with fewer zones.

To determine which Koopman modes are most prevalent in the thermal behavior of the building, the spectrum of the Koopman operator was calculated from observations of zone temperatures. The magnitude of the spectrum for the detailed EnergyPlus model is illustrated in Figure 2.

To interpret Figure 2, each column of the figure corresponds to a Koopman mode at a particular frequency and each row represents a thermal zone. Horizontal bands

Table 1: Features of the EnergyPlus model

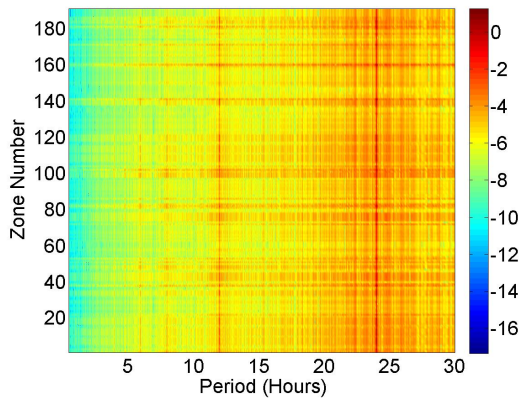|	# of Zones	Area (sqft.)
Total Building	191	80,500
Offices	94	15,000
Clean Room	14	10,000
Laboratories	49	21,000
	Quantity	
Modelled Walls	2247	
Modelled Windows	478	

Figure 2: (Color Online) Koopman spectrum for the zones of the detailed EnergyPlus model (Periods larger than 30 hours are not illustrated).

Table 2: Statistics of the Koopman modes that are largest in magnitude.

Period (Hours)	Maximum	Minimum	Mean
8760	3.459	1.000	2.040
24	2.055	0.121	0.626
12	0.788	0.022	0.136
8	0.371	0.004	0.048
6	0.221	0.006	0.029

in the spectrum denotes that the temperature behavior of a particular zone has spectral content across multiple Koopman modes while vertical bands indicate that a large group of zones are influential to a particular Koopman mode. Within the spectrum, the modes with the largest magnitude correspond to the 8760 hour, 24 hour, 12 hour, 8 hour, and 6 hour period. Table 2 compares the magnitudes of these modes. The modes capture features of oscillations of zone temperature occurring at different time scales. Since the temperature of the model studied is only affected by weather, the longest duration mode reflects temperature trends that occur seasonally (e.g., higher zone temperatures during the summer/lower zone temperature during the winter) while the remaining modes identify temperature patterns at daily and hourly time scales.

Comparing the amplitudes and phases of zones within each mode highlights elements of the design of the building. In Figure 3, the distribution of amplitudes and phases of the 5 modes of Table 2 are shown, and in Figure 4, the amplitudes and phases of zones from the longer duration modes are shown spatially against a plan view of the second floor of the building.

In these figures, modes at longer time scales contain smaller variations in phase, but have large variations in
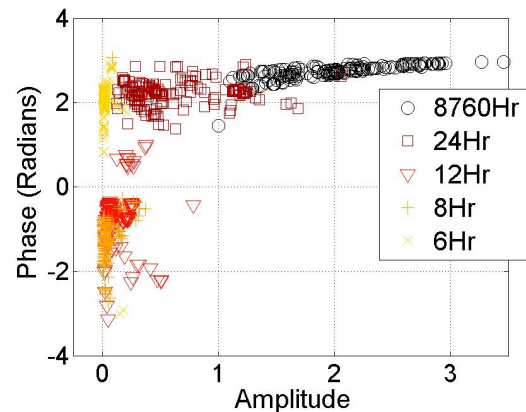

Figure 3: (Color Online) Amplitude and phase of zones for the five modes considered. Adjacent zones with similar amplitudes or phases are grouped together when creating zoning approximations.

amplitude. Looking at modes on faster time scales, the opposite occurs as modes contain large variations in phase with smaller changes in the values of amplitude between zones. These patterns reveal some dominant features and effects of the simulated weather and building orientation on zone temperature.

Within the mode of yearlong duration, the phases of zones are similar in magnitude (compared to modes of shorter duration) since heat transfer at this time scale occurs more slowly compared to the time constant of the thermal mass of the building, but the amplitude of zones vary greatly since there are different amounts of glazing from windows and insulation between different areas of the building. The zones corresponding to the more heavily insulated research laboratories have a lower amplitude of temperature oscillation compared to the heavily glazed naturally ventilated offices. In the modes reflecting building dynamics at faster time scales, there are large variances in phases of zones as these modes reflect orientation driven short term events such as early morning and late afternoon sun exposure that can cause a sudden surge in heating in a zone for several hours depending on which side of a building the zone is located on.

Using characteristics of these modes, one can better understand when zones may behave similarly at one time scale but differently at another. By exploiting similarities of adjacent zones occurring among all influential modes, a zoning approximation can be made that best preserves the temperature behavior of the original model.

In comparing these modes, adjacent zones are grouped together, with respect to having similar amplitude or phase within a mode, if their distance from each other is within a pre-specified tolerance r. The following metrics were
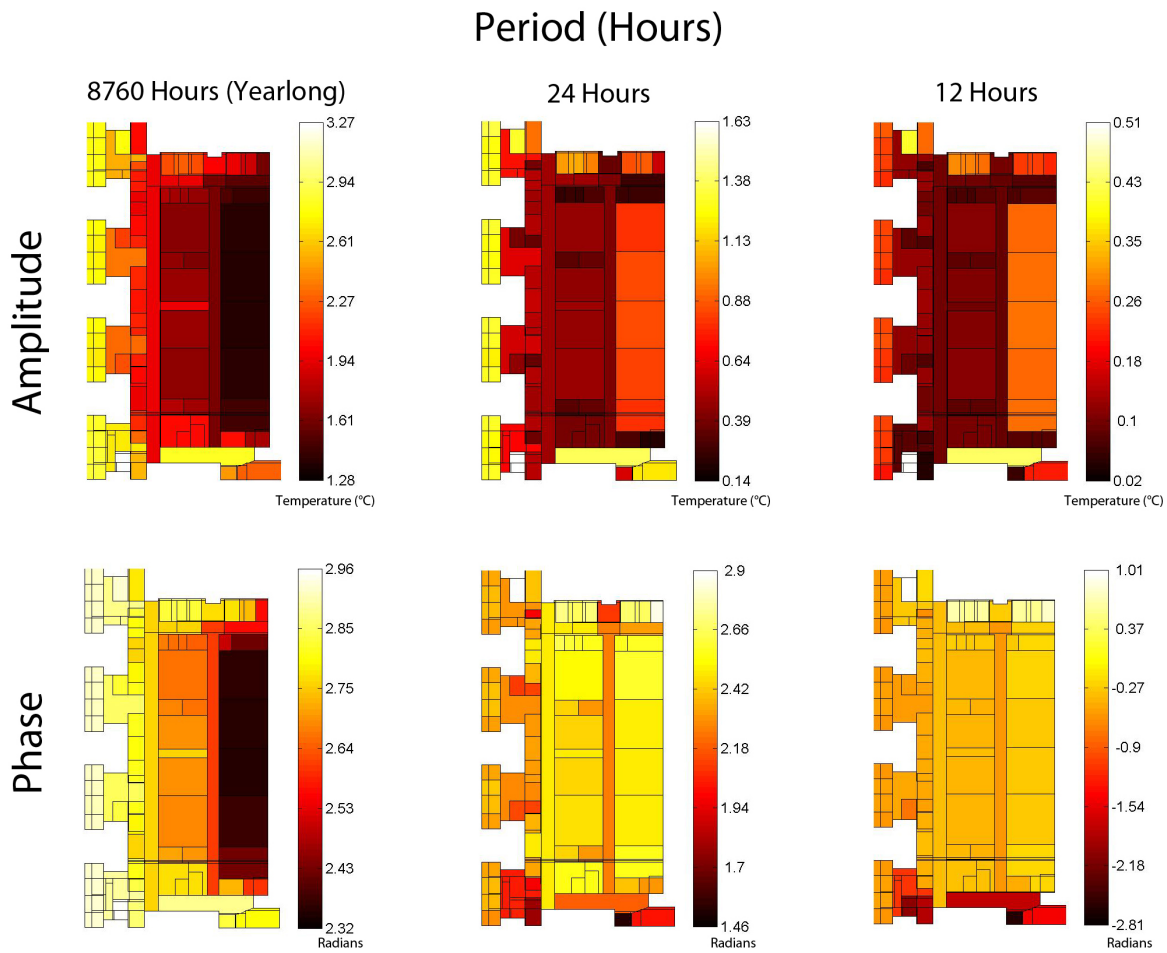
Figure 4: (Color Online) Amplitudes (top row) and phases (bottom row) of zones from the three Koopman modes that are largest in magnitude plotted against a plan view of the 2nd floor of the building. (Note differences in magnitude of color scale between images)

used for comparing the amplitudes and phases of zones: given two zones i and j and a maximum allowable difference in magnitude within a particular mode k, if

$$\left| \|v_k(x_i)\| - \|v_k(x_j)\| \right| < r, \tag{5}$$

zones i and j are grouped together and treated as similarly behaving with respect to their amplitude within the mode considered.

For the phase of two zones i and j and a maximum allowable difference in magnitude within a particular mode k, if

$$\left| \angle(v_k(x_i)) - \angle(v_k(x_j)) \right| < r, \tag{6}$$

zones i and j are grouped together and treated as similarly behaving with respect to their phase within the mode considered.

considered.

When creating a zoning approximation, two zones are lumped together if they are grouped with respect their amplitude and phase across all modes. Because the amplitudes and phases of zones are distributed differently across the Koopman modes considered, two adjacent zones may not be grouped based on amplitude and phase across all modes for a particular value of r. Figure 5 illustrates the sensitivity of the grouping of zones as the value of r is varied.

The value of r was normalized across the phases and amplitudes of different modes, so that when a zoning approximation is created, the amplitudes and phases across all modes are weighted equally. The procedure of creating zoning approximations from an EnergyPlus model using Koopman modes is:
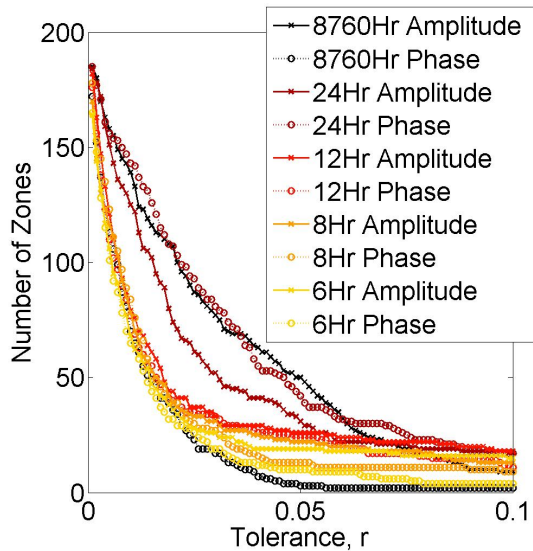
Figure 5: (Color Online) The sensitivity of the reduction in the number of zones as the magnitude of r is increased.

Table 3: Change in model predictive capability of heating and cooling loads as the number of zones are reduced.

# of Zones	Heating (kWh)	Cooling (kWh)	% Error
191	384526	113144	0.0%
176	384645	113094	0.01%
154	384631	113124	0.02%
135	384504	113142	0.00%
112	384471	112844	0.07%
103	384442	112801	0.09%
95	383852	112211	0.32%
89	383781	112207	0.34%
77	383373	111808	0.50%
72	382717	111951	0.60%
60	382431	111655	0.72%
32	374011	106897	3.37%
10	346216	86128	13.13%

1. Simulate full-order EnergyPlus mode outputting observables of interest (in this case zone temperature).

2. Calculate Koopman modes by projecting observables onto eigenfunctions of the Koopman operator.

3. Combine adjacent zones with Koopman modes of similar amplitude and phase at frequencies of interest.

Using this procedure, numerous zoning approximations, with varying degrees of simplification, were created by adjusting the value of *r*, and simplified EnergyPlus models with a reduced number of zones were created. The performance of these models will now be discussed.

COMPARISON OF ZONING APPROXIMATIONS

A number of simplified models were created by increasing the tolerance, *r*, used in determining if two adjacent zones behave sufficiently similar to be lumped together. When zones are lumped, shared walls are modelled as internal zone objects with thermal mass. To gauge the impact of reduced zoning on model predictive ability, the effect of an HVAC system was simulated, using the "Ideal Loads Air System" in EnergyPlus, where each zone is ideally heated/cooled, and the temperature of each zone is controlled to 20°C throughout the simulation. Accuracy of a simplified model is determined by comparing its prediction of HVAC energy consumption with that of the original detailed model. Additionally, the effect of neglecting the thermal mass from shared walls of reduced zones was also investigated. This was done to reflect stan-

dard practice, as shared walls are not commonly captured in building modelling programs when multiple rooms are treated as a single zone in reduced models. The accuracy of simplified models with varying level of zoning refinement are shown in Figure 6 and Table 3.

The original detailed model contains 191 zones. As the tolerance increases when comparing amplitudes and phases of Koopman modes, more adjacent zones are determined to be sufficiently similar to each other, reducing the total number of zones. When the shared walls of lumped rooms are neglected, accuracy decreases more quickly as illustrated in Figure 6. As the number of zones reduces, the calculated HVAC energy decreases as shown in Table 3. This will always occurs when zones are lumped since the air within the building becomes more well-mixed. In Figure 6, the error in prediction is inversely proportional to the number of modelled zones.

Between the models, there is very little loss in predictive ability when the shared walls of lumped rooms are treated as internal masses in the reduced models. If these internal walls are not modelled, accuracy of the reduced model diminishes at a much faster rate. To get an impression of how a reduced model is zoned, the floor plans of the 191 zone, 112 zone, and 60 zone models are illustrated in Figure 7.

In the reduced models, one can see how using Kopman modes preserves features of the floor plan of the building that were previously seen in Figure 4. Despite the differences in zoning between the original model and 60 zone model, the error of the reduced model is less than 1% for the conditions simulated.

CONCLUSIONS

In this paper, a detailed building energy model was analyzed using the Koopman operator in order to identify,
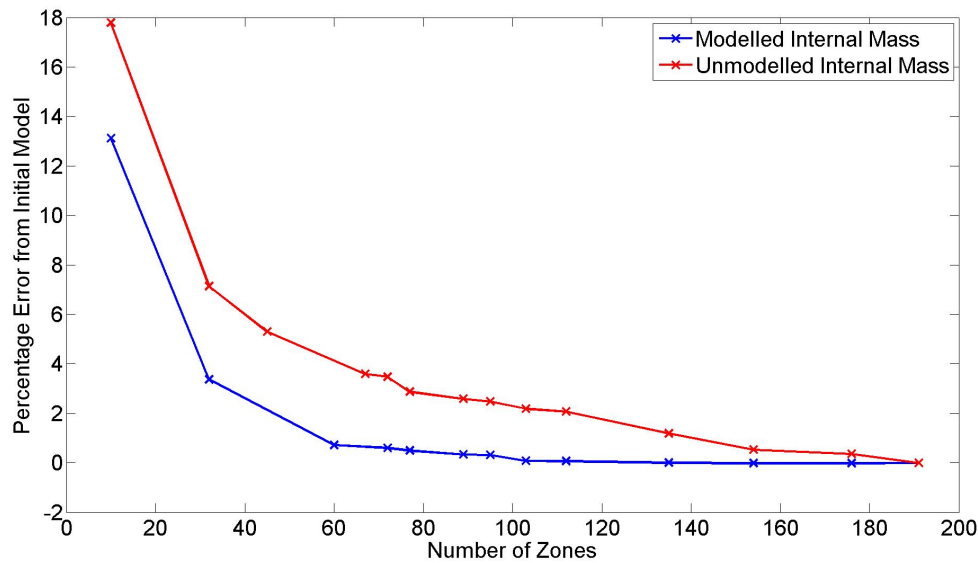
Figure 6: Simplified model accuracy (relative to initial detailed model) as the number of zones are reduced.

and systematically develop, zoning approximations based off of observations of zone temperature. The goal of these approximations is to reduce the complexity of the model while minimally impacting model accuracy. Through calculating Koopman modes, influences of the simulated weather on zones can be seen at different time scales, and by identifying similarities across these time scales, models containing a reduced number of zones were created which attempted to best preserve the temperature behavior of the original detailed model.

In the model studied, the number of zones were reduced from 191 zones to 32 zones with a 3.3% error in prediction (7.1 % if thermal mass from internal walls of lumped zones are neglected). Unfortunately, the detailed model had to be simulated once in order to gather trends on the temperature histories of zones with which to analyze the system. From the zoning approximations generated through this method, several observations were made on types of simplifications that had little impact on accuracy. These simplifications can potentially be applied when creating other models without requiring a detailed model to be simulated. Some guidelines to observe which can help maintain model accuracy are:

- When merging zones, the thermal mass of unmodeled walls should be captured

- Zones containing exterior surfaces should not be merged with zones that do not contain exterior surfaces

- Perimeter zones that are merged should have similar

surface orientations and window areas

- Zones containing a small volume and surface area can be merged with a much larger adjacent zone with little change loss of accuracy
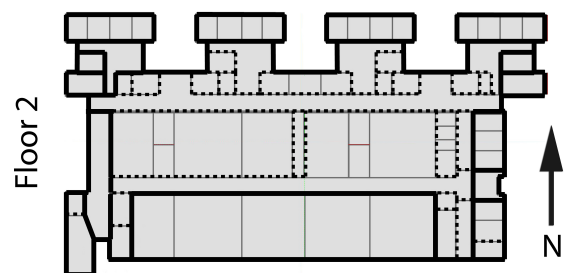

Figure 7: Boundaries of two zoning approximations compared to the original model zone layout (second floor of the building shown). Gray lines indicate zone boundaries of original 191 zone model. Thick solid lines indicate zone boundaries of a 60 zone approximation and dotted lines indicate additional boundaries of a 112 zone approximation.

An obstacle encountered during model reduction described in this paper is that a detailed model must first be created in order to establish a baseline for model accuracy. However, creating a detail model in this way takes a significant amount of time, is more computationally expensive, and often amplifies uncertainty (when the number of parameters in the model increases). Accurate model

simplification described in this paper addresses the computational time required by reducing the number of constructions in a building model. In addition to this, when similar zones are aggregated, less parametric information is needed to describe the building and hence reduce uncertainty in its prediction.

ACKNOWLEDGEMENTS

NOMENCLATURE

F	nonlinear function
x	state space variables
x_i	i-th state space variable
t	sampling instance
M	high dimension manifold
$\mathbb{R}$	set of real numbers
U	Koopman operator
g	observation function
ψ_k	k-th Koopman eigenfunction
λ_k	k-th Koopman eigenvalue
v_k	k-th Koopman mode
k	eigenvalue/eigenfunction index
$\mathbb{C}$	set of complex number
r	maximum allowable difference in magnitude

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