A Note on the Path-Dependence of the *J*-Integral Near a Stationary Crack in an Elastic-Plastic Material With Finite Deformation

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In this technical brief, we compute the J-integral near a crack-tip in an elastic-perfectly-plastic material. Finite deformation is accounted for, and the apparent discrepancies between the prior results of the authors are resolved. [DOI: 10.1115/1.4006255]

1 Introduction

Recently Carka and Landis [1] have demonstrated the path-dependence of the *J*-integral [2] in elastic-plastic materials under plane strain conditions. These calculations, using the assumptions of linear kinematics, show that the value of the *J*-integral for a circular contour on the crack-tip is approximately 18% lower than its far field level in an elastic-perfectly-plastic material with Poisson's ratio in the range of most metals. Within a full finite deformation setting, McMeeking [3] showed that *J* was markedly path-dependent very close to the crack tip where the change in geometry associated with crack-tip blunting has a significant effect. In fact, McMeeking's results suggest that *J* goes to zero for a contour approaching the tip of a crack that was sharp prior to blunting. Due to the manner in which McMeeking generated and plotted values for *J* versus the size of the path-integral contour, some questions may arise about the consistency between the results of McMeeking [3] and Carka and Landis [1]. This note attempts to make the connection between these two works and bring closure to the issue.

2 Results and Discussion

The primary differences between the calculations performed in Refs. [1,3] include, (a) the description of the kinematics and the associated issues for describing the constitutive behavior, (b) the method for applying the far-field boundary conditions, (c) the type of finite-elements and number of degrees of freedom in the plastic zone used, and (d) the method for computing *J*. In this note, the finite deformation formulation and finite-element implementation of McMeeking [3] and McMeeking and Rice [4] is used, the far-field boundary conditions due to Carka et al. [5] are applied, nine-noded quadrilateral elements with full integration of deviatoric strains and reduced integration of hydrostatic strains are implemented, and the domain integral method is used to compute *J*.

The results for the *J*-integral as a function of the radius of the circular contour are plotted in two ways in Fig. 1, and the crack tip opening displacement as a function of the far-field applied *J* is plotted in Fig. 2, all for elastic-perfectly-plastic materials with Poisson's ratio of 0.3. Figure 1(*a*) plots the value of *J*, normalized by the far field applied value, as a function of the radius of the integration contour in the initial undeformed configuration normalized by the plastic zone size, $R_p = (K_I/\sigma_0)^2/3\pi$. Here K_I is the far-field applied mode I stress intensity factor, which is related to the far-field applied *J* value as $J_\infty = K_I^2(1-\nu^2)/E$, and σ_0, E, and ν are the yield strength, Young's modulus, and Poisson's ratio of the material.

In addition to the size of the plastic zone, the consideration of finite deformation introduces a second length scale into the small-scale yielding problem, namely the crack tip opening displacement δ_{tip}. The crack tip opening displacement can be approximated as $\delta_{tip} = 0.6 J_\infty/\sigma_0 = 1.8\pi(1-\nu^2)(\sigma_0/E)R_p$. The prefactor of 0.6 has a mild dependence on σ_0/E as is plotted in Fig. 2. Note in Fig. 2 that in the limit as $\sigma_0/E \to 0$, $\delta_{tip}\sigma_0/J_\infty \to 0.61$, which is the result from the linear kinematics computations in Ref. [1]. Figure 1(*a*) shows that as the size of the

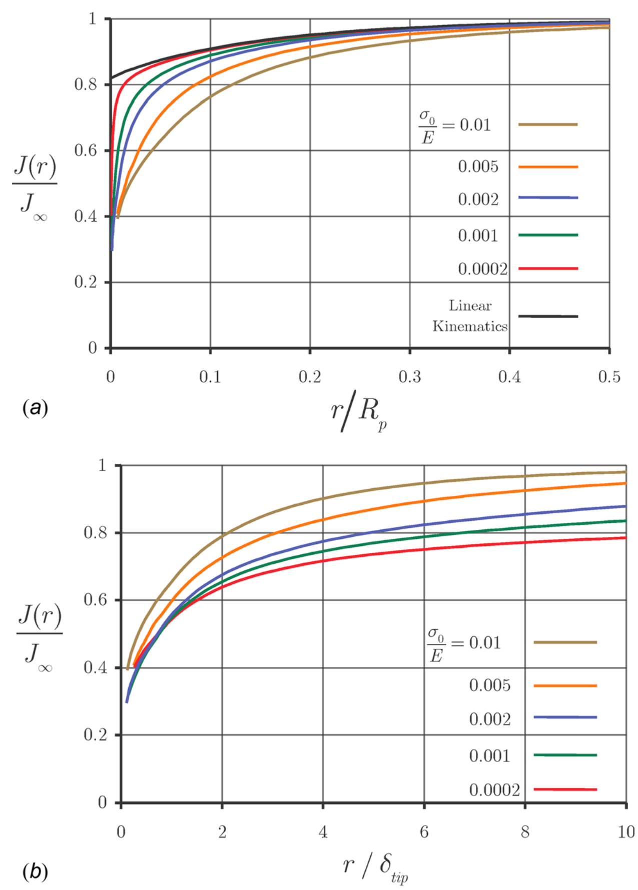

Fig. 1 The *J*-integral normalized by its far-field value as a function of the radial distance of the contour from the crack-tip in the undeformed configuration. All results are for elastic-perfectly-plastic materials. (*a*) With the contour distance normalized by the characteristic size of the plastic zone. (*b*) With the contour distance normalized by the crack-tip opening displacement.

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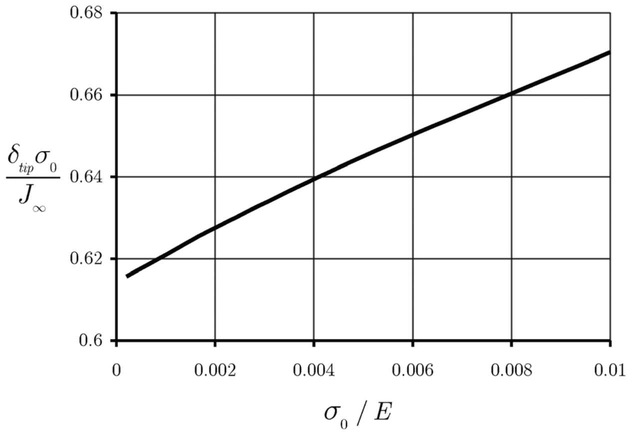

Fig. 2 The normalized crack-tip opening displacement as a function of the yield strength to Young's modulus ratio in an elastic-perfectly-plastic material

integration contour approaches δ_{tip}, the finite deformation results begin to significantly deviate from their linear kinematics counterpart. Figure 1(*b*) also plots the normalized J values as a function of the contour size but with the contour radius normalized by the crack opening displacement. These results confirm McMeeking's finding that J appears to vanish as the radius of the contour shrinks to the crack tip. Away from the crack tip, there appears to be disagreement with the results reported in Ref. [3]. However, this is not the case. In Ref. [3] the results for J were reported for three different contour radii in the initial undeformed configuration, and these radii were normalized by the diameter of the blunted crack tip b, which included both the crack opening displacement δ_{tip} and the initial blunt notch diameter b_0. Values for J/J_∞ were then computed as the loading progressed and consequently as the crack tip diameter, $b = b_0 + \delta_{tip}$, increased with respect to the location of the contours. Since each contour is located in entirely elastic material when the loading begins, the value for J/J_∞ at such a point in the calculation must be unity. Furthermore, since the position of the contour is normalized by b and not δ_{tip}, the point on the ordinate $J/J_\infty = 1$ resides at multiple different finite distances from the crack-tip on the abscissa depending on the location of the contour, instead of infinitely far from the crack-tip as would be the case for the fully self-similar solution. The results in Ref. [3] do show that as plasticity progresses J decreases, and very close to the crack tip it drops precipitously towards zero. What is perhaps misleading about Fig. 9 in Ref. [3] is that only the very last few points on each of the three branches are close to the self-similar steady-state solution. Hence, the appearance in Ref. [3] that J is essentially path-independent outside of a few crack-tip opening displacements is an artifact of the method used to generate and plot the results. Figures 1(*a*) and 1(*b*) of this note plot only points from the self-similar solution and demonstrate how the crack-tip blunting associated with finite deformation leads to deviations from the linear kinematics results for the path-dependence of J.

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